Analyzing a complex system

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Abstract

The aim of this work is to explore some ways to draw out information from the solutions of a dynamical system having two kinds of complexity, a high number of interacting freedom degrees and time varying coefficients. Some geometrical properties conferred by the system to the phase space are used to split up the system in simple elements and to analyze the symmetries. The model taken as an example concerns the dimethylsulfide (DMS) cycle. It consists of an eight-dimensional dynamical system with periodic coefficients. This problem asks more mathematical questions than it is possible to answer given what we know at the moment. But some features of the behavior of the solutions can be analyzed.

The dimethylsulfide (DMS) molecule dissolved in sea water evaporates under some conditions and helps to supply most of the cloud condensation nuclei in the atmosphere. So the DMS cycle of ecosystems contributes to scatter and absorb incoming solar radiation and to moderate anthropogenic forcing of climate. This field gives rise to a broad interest and to a large number of papers, but the magnitude of the climate feedback of the DMS is difficult to appreciate.

The first part of this work is devoted to the construction of a model of the biogeochemical cycle of DMS based on works of A. J. Gabric & al.[1]. The variables of an eight-dimensional mathematical model are concentration of phytoplankton, bacteria, zooflagellates, large protozoa, micro and mesozooplankton, dissolved inorganic nitrogen, dissolved dimethylsulfonio-propionate (DMSP) and dissolved DMS. The air-sea exchange of DMS depends in a complex way on the wind velocity and on the sea surface temperature which is a function of time.

At first, the asymptotic behavior of solutions is analyzed with the data of biologists and the interactions between the populations are compared to reduce the number of dimensions of the dynamical system

Then, the equation of an invariant manifold of an associated constant coefficient equivalent system is computed in a very simple way using differential geometry results. This manifold is periodically crossed by the solutions and is involved in the structure of the attractor. On the other hand, the manifold may bring to light some symmetries of the solutions.

The respective influence of other set of variables could be studied by this method.

1. Gabric A. J., Gregg W., Najjar R., Erickson D., Matrai P., 2001. Modeling the biogeochemical cycle of dimethylsulfide in the upper ocean: a review. Chemosph. Global Change Sc. 3: 377-392.