Is the wave turbulence observed in elastic plates related to "weak turbulence"?

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It has been observed recently that wave turbulence can develop in vibrated elastic plates (Mordant PRL 2008, Boudaoud et al. PRL 2008). A statistical theory of wave turbulence (so called weak turbulence theory or WTT) exists for more than half a century and has been applied in a large variety of systems ranging from condensed matter physics to astrophysics. In particular, it has been applied to the case of elastic plates (Düring et al. PRL 2006). The experimental single point spectra are not in agreement with the WTT predictions. The measured frequency spectra are steeper than the prediction and their scaling with the average injected power is also not that predicted by the WTT. The reasons for this disagreement with the WTT could be related to the level of non linearity, finite size effects or dissipation (if not restricted to small scales).

P. Cobelli, P. Petitjeans, A. Maurel (ESPCI, Paris) and V. Pagneux (Univ. du Mans) developed a Fourier transform profilometry technique that we applied to the elastic plate turbulence (Cobelli et al. PRL 2009). It allows us to measure the deformation of the plate over a significant part of the surface of the plate. The time resolution is provided by the use of a high speed camera. In this way, movies of the plate deformation can be recorded. It allows us to get the full space and time Fourier spectrum and thus to probe in much more details the structure of the wave turbulence than with single point spectra. We observe in particular that energy is indeed localized on a surface in the 3D (k_x, k_y, ω) space as expected from waves. The observed non linear dispersion relation is close to the linear dispersion relation which confirms a weak non linear coupling of the waves. The shift between the two dispersion relations is seen to increase with the forcing. The thickness or the dispersion relation (energy surface in the (k_x, k_y, ω) space) is seen to also increase with the forcing. All these features are in qualitative agreement with the predictions of the WTT and thus do not provide explanations for the observed disagreement with the theory. Finite size effects are also observed but vanish as the forcing amplitude is increased. The FTP technique is a very promising tool for a extensive quantitative analysis of the wave turbulence observed in elastic plates. It can also be applied to turbulence of fluid surface waves. Preliminary studies show that the development strong linearities is observed in that case. The ability of the FTP technique to provide the space-time dynamics of surface deformations makes it a specially suited tool for the analysis of wave turbulence.