

A matched filter for chaos: the missing piece for chaos communications

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In conventional communication systems, a matched filter provides optimal receiver performance in the presence of noise. As such, matched filters are highly desirable, yet they are practical only when a relatively small number of basis functions are used to encode information. For communications using chaotic waveforms, it is generally assumed that the unpredictable and nonrepeating nature of chaos precludes the use of a matched filter; consequently, it is widely accepted that using chaos for communications results in lower performance capabilities compared to conventional, nonchaotic systems. Here, we show this assumption is not necessarily true. We report the construction and operation of a novel chaotic electronic oscillator that admits a simple matched filter. The audio-frequency circuit, which contains both analog and digital components, is modeled by a hybrid dynamical system including both a continuous differential equation and a discrete switching condition. Surprisingly, an exact analytic solution for the system can be written as the linear convolution of a symbol sequence and a fixed basis function, similar to conventional communications waveforms. Waveform returns sampled at switching times are conjugate to a shift map, effectively proving the circuit is chaotic, and the analytic solution accurately reconstructs a measured waveform, thereby validating the circuit model. A matched filter for the basis function is derived in the form of a delay differential equation. An experimental realization of the matched filter is implemented in a simple analog circuit. The filter is used to detect the symbolic dynamics of the oscillator waveform, and an analytic bit-error rate is found to be comparable to binary phase-shift keying (BPSK). Scaled to higher frequencies, this oscillator has potential application in Hayes-type chaos communications where a message signal is encoded in the symbolic dynamics via small perturbation control. The discovery of a practical matched filter finally provides a coherent receiver to complement the elegant encoding in such systems.