Characterising time series dynamics with complex networks

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The application of complex network structure for the analysis of time series has recently led to several new approaches to quantify dynamical behaviour in nonlinear systems. In general these methods construct some sort of network from the time series by mapping dynamical states in the underlying system to individual nodes and drawing links between similar nodes. In particular, one of these methods* has shown considerable promise by providing a classification for dynamical behaviour. By measuring the relative frequency of occurrence of different motifs this method has been shown to be able to differentiate between low-dimensional chaos (one positive Lyapunov exponent), hyper chaos, periodic, quasi-periodic and noise periodic dynamics.

By applying this method to nonlinear time series models we show how this method can be extended to short and noisy time series, and can be used to both evaluate and qualitatively describe the performance of these models. We build nonlinear models (we use a radial basis model structure, but the choice is arbitrary) from time series data and then evaluate features of the complex networks structures for time series simulations produced by these models and for the original data. In cases were the original data was sufficient to make a meaningful assessment of the network structure we can determine which models are qualitatively good models. In cases were the original data is insufficient we can use the performance of the models as a surrogate and make a meaningful estimate of the various possible alternatives.

We apply the method to a short ecological time series and an ensemble of long time series of sustained musical tones. For the ecological time series (annual populations of Canadian Lynx) we find the previous pronouncements of chaos in this system are premature. For the tone data (pure tones on a standard B♭ clarinet) we show strong evidence for bounded aperiodic dynamics which is not consistent with low-dimensional chaos. Further support for this conclusion can be obtained from surrogate time series methods and some of the more usual nonlinear time series measures. We also observe that (for the clarinet data) the models with the "right" dynamics are also the models that sound "right".

*X. Xu, J. Zhang and M. Small. "Superfamily phenomena and motifs of networks induced from time series." Proceedings of the National Academy of Sciences of the United States of America 105 (2008): 19601-1960