Arbitrary order Hilbert spectral analysis : a new tool to analyze the scaling complexity of time series, application to turbulence data

Francois Schmitt[1], Yongxiang Huang[1,2], Zhiming Lu[2], & Yulu Liu[2]

[1] Laboratory of Oceanology and Geosciences, CNRS-University of Lille 1, France
[2] Shanghai Institute of Applied Mathematics and Mechanics, Shanghai University, China
francois.schmitt@univ-lille1.fr

Empirical Mode Decomposition (EMD) is an analysis technique introduced by Norden Huang in 1998; it was imagined to decompose a complex time series into a sum of modes, each one being narrrow banded. This method is fully data-driven, and is suitable for nonlinear and nonstationary time series. Since its introduction this method has been applied in more than 1000 papers, in many fields of natural sciences including oceanic and atmospheric sciences, climate studies, mechanical engineering, biomedical and biological sciences, among others. It has been completed by Hilbert Spectral Analaysis (HSA), a method involving Hilbert transform to characterize time series fluctuations in an amplitude-frequency space.

Here we generalize this approach in order to characterize the scaling intermittency of complex time series in an amplitude-frequency space. The new method is a arbitrary order Hilbert spectral analysis. As a first step the method is applied to fractional Brownian motion, and then to homogeneous turbulence data and chaotic and nonlinear signals.

We show that Hilbert spectral analysis can be used to recover the Kolmogorov -5/3 inertial range; we obtain a 2D amplitude-frequency representation of the pdf $p(\mathcal{A},\omega)$ of turbulent fluctuations with scaling trend. We obtain multifractal scaling exponents in amplitude-frequency space and show that they are close to the ones in real space, despite the quite different approaches used in both cases. We find that the new methodology provides a better estimator than the classical structure functions.

We then investigate the effect of a periodic component on both structure functions and the Hilbert approach, and find that the former one is strongly influenced by the periodic component, whereas the latter can constrain such effect in an amplitude-frequency space. This shows the usefulness of this new method for general scaling processes and especially for time series possessing energetic large scales.

This new approach is able to characterize the multi-scale properties of the fluctuations of nonlinear time series. It is likely to have many different applications for data analysis of nonlinear, chaotic and complex time series.

Huang Y., F. G. Schmitt, Z. Lu, Y. Liu, EPL 84, 40010, 2008

Huang Y., F. G. Schmitt, Z. Lu, Y. Liu, EPL 86, 40010, 2009

Schmitt FG, Y Huang, Z. Lu, Y. Liu, N. Fernandez, Journal of Marine Systems 77, 473-481, 2009

Huang Y., F. G. Schmitt, Z. Lu, Y. Liu, Journal of Hydrology 373, 103-111, 2009