On the unique reconstruction of a signal from its recurrence plot

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Recurrence plots are two-dimensional representations of high-dimensional trajectories of dynamical systems. Patterns in recurrence plot carry information on the underlying trajectories and can be studied and analyzed for detection and classification purposes. From the literature it is known that a recurrence plot determines its underlying trajectory up to isometry. Here we consider trajectories that are obtained from a one-dimensional signal with the time-delay embedding method. We address the question to which extent a recurrence plot determines the underlying signal. First we show that a recurrence plot determines the power spectrum of this signal. Then we provide conditions on the embedding dimension and the time-delay which imply uniqueness of the underlying signal (up to a sign factor). A worked example from EEG analysis illustrates how this theory allows one to understand the limitations that apply to the interpretation of a recurrence plot. We consider a measured EEG signal containing a so-called Mu rhythm, i.e. exhibiting an m-shaped morphology with frequencies between 8 Hz and 12 Hz. We show that for some values of the embedding dimension and time-delay, another signal with a different morphology can be constructed which yields the same recurrence plot. This induces ambiguity in the interpretation of the associated recurrence plot. We also show how to avoid this phenomenon by appropriately choosing the embedding dimension and time-delay parameters to guarantee uniqueness of the corresponding pattern in the recurrence plot.