Modal interactions in thin structures: some experiments on non-linear vibrations of spherical shells and percussion musical instruments

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Structures with a thin geometry, like beams, plates and shells, can exhibit large amplitude flexural vibrations, whose magnitude is comparable to the order of their thickness. In those cases, typical non-linear behaviors can be observed. Among others, the response of the structure can exhibit multiple stable solutions that lead to jump phenomena and significant non-linear energy transfers between modes, associated to quasi-periodic and chaotic motions. Those phenomena are encountered in various engineering structures, from macro-scale structures such as helicopter blades to micro and nano-electromechanical structures (M/NEMS). They are the main physical source of the particular sound of percussion musical instruments such as gongs and cymbals.

The purpose of the present study is to present some experiments on non-linear vibrations of percussion musical instruments and similar circular plate and shell structure, in order to give insights in their non-linear vibratory behavior and to explain some features of their particular sound. In a first part, a chinese gong excited by a harmonic force in the vicinity of one natural frequency enables to exhibit a generic route to chaos observed in those shell-like structures. For low excitation levels, periodic motions are observed, with a motion dominated by one master natural mode. Then, a first bifurcation lead to a quasi-periodic regime where several vibration modes exchange energy with one another. This specific vibratory regime appears when internal resonances (*i.e.* specific algebraic relations between the natural frequencies) between modes are present. Finally, a second bifurcation is observed, leading to a chaotic motion.

In a second part, a detailed study of some non-linear forced vibration regimes involving internal resonances is proposed. Two cases are studied: a 1:1 internal resonance in a circular plate and a 1:1:2 internal resonance in a shallow spherical shell. In both cases, because of the rotationally symmetric geometry of these structures, all modes with nodal diameters appear in pair, with both modes associated to the same natural frequency (leading to the 1:1 resonance) and their modal shapes differing only by the angular position of their nodal diameters. In the case of the spherical shell, the 1:1:2 resonance is observed between an axisymmetric mode and two companion asymmetric modes of half its frequency. The amplitudes of the modes, as measured by accelerometers, are shown as a function of the excitation frequency, its amplitude being kept constant. Various coupled regimes are exhibited, leading to jump phenomena and traveling wave motions. Movies obtained with stroboscopic lighting are also available. The obtained frequency response curves are successfully compared to reduced order models composed of two or three non-linear oscillators with coupled quadratic and cubic non-linear terms, that help understanding the observed particular coupled vibratory regimes.