Synchronization phenomena in networks of neuron models

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Synchronization phenomena arise within many natural or artificial interaction networks. In this work we consider Hindmarsh-Rose oscillators modeling an individual neuron behavior and connected in a network with adjacency matrix $\{c_{ij}\}$ (1). The neurons are coupled by nonlinear functions (2) modeling chemical synapses.

$$\begin{cases} \dot{x_i} &= ax_i^2 - x_i^3 + y_i - z_i - \sum_{j=1}^n c_{ij} h(x_i, x_j) \\ \dot{y_i} &= (a+\alpha)x_i^2 - y_i \\ \dot{z_i} &= \epsilon(bx_i + c - z_i) \end{cases} \qquad i = 1, \ldots, n \tag{1}$$

$$h(x_i, x_j) = g_{\mathrm{syn}}^{(n)} \frac{(x_i - V)}{1 + \exp(-\lambda(x_j - \Theta))} \tag{2}$$

The HR model exhibits most of the behaviors observed in the case of real neurons, such as spike firing or bursting. The bursting motion consist of successive series of action potentials separated by slow periods.

The most studied kind of synchronization is the so called complete synchronization, which means that all the nodes of the network share the same behavior at the same time. In this work we study the condition of complete synchronization in networks of coupled neuronal models. We show that in order to obtain synchronization, all the nodes must have the same in-degree. The minimal coupling force needed to synchronize a network depends only on the in-degree of the nodes as a power law.

The necessary condition for complete synchronization is quite restrictive and biologically unrealistic, that is why we consider another type of synchronization phenomenon, called burst synchronization.

An oscillator network presents burst synchronization if the n oscillators of the network fire bursts starting all at the same time. If the complete synchronization is easy to detect, no algorithm exists to detect burst synchronization. Therefore, in this work, we propose an algorithm of burst synchronization detection within networks. Our algorithm can be decomposed in four main steps. In order to detect burst synchronization, bursts of different neurons must be matched. To do this, one needs to determine the start time of each burst, and before detecting bursts, spikes must be detected first. This algorithm is then applied to different kinds of networks topologies for which we study the minimal coupling force needed to obtain burst synchronization.