Weak temporal ratchet effect by asymmetric modulation of a laser

Cristina Elena Preda, Bernard Ségard, and Pierre Glorieux
Laboratoire de Physique des Lasers, Atomes et Molecules, CNRS (UMR 8523) CERLA, Université de Lille I, 59655 Villeneuve d’Ascq, France

Received March 21, 2006; revised May 5, 2006; accepted May 8, 2006; posted May 15, 2006 (Doc. ID 69194); published July 10, 2006

Subjecting lasers to triangular modulations of the pump produces phenomena that drastically depend on the symmetry of the triangle. For instance, with slow up-rising triangles, a laser below threshold on average may deliver coherent pulses, while it does not deliver coherent pulses with fast up-rising triangles. This effect, which requires the presence of fluctuations in addition to the modulation, is reminiscent of the thermal ratchet with which a directed motion is extracted from a stochastic medium. © 2006 Optical Society of America

OCIS codes: 140.0140, 140.3480, 270.2500.

Extracting energy from Brownian motion by subjecting a fluctuating system to suitably tailored perturbations has been a long-standing subject of investigation, starting from Feynman’s ratchet to the more recent quantum theory of optical ratchet effects in cold atom clouds. A typical approach is to use some periodic asymmetric perturbation to induce a global drift in a system without a gradient via the interplay of this perturbation and of the fluctuations. In the typical ratchet, the perturbation is much smaller than the fluctuations. Here we explore the reciprocal situation in which the perturbation dominates the fluctuations, and our system uses pure temporal dynamics, while space and time dynamics are coupled in the classical ratchet. In the latter system, a change in the sign of the perturbation induces a similar change in the direction of the energy flow. In the present situation, the perturbation symmetry governs the existence or nonexistence of this energy flow. However, as for the ratchet, the observed phenomena do require the simultaneous presence of fluctuations and asymmetric modulation.

In this work a laser below threshold is subjected to an asymmetric change of its excitation conditions, via a triangular time-periodic (sawtooth) modulation of its pump parameter such that the average value of the pump is close to but remains below threshold. Positive and negative sawtooth modulations have drastically different effects. For instance, a slow up-rising sawtooth (called “positive”) does trigger laser action, while a fast up-rising sawtooth (negative) does not, although one would expect the opposite, since the negative sawtooth initially provides much stronger pump power. Because a positive sawtooth triggers laser action, a directed coherent energy flow is delivered by the laser, while incoherent emission only is obtained with the negative one. In this Letter we demonstrate experimentally and explain, on the basis of a simple model, that a directed energy flow is obtained only if perturbations with suitable asymmetry and fluctuations are simultaneously applied to the laser.

Because here we use asymmetric modulations, the behavior of the laser is not simply understood in the framework of previous investigations regarding lasers as nonlinear dynamic systems, since this point of view is relevant for sinusoidal modulations or symmetric triangular sweeping. It is classical that, when subjected to a slow periodic excitation, systems adiabatically follow the changes induced by the modulation, while for fast modulation, they respond only to the time-averaged value of the perturbation. Between these two limits, when the perturbation period is similar to relevant time scales for the system, e.g., the relaxation oscillation period, switch on time, etc., there is a possibility of rich dynamic effects. Sufficiently strong periodic modulations in this range are known to generate nonlinear effects including period-doubling bifurcations leading to chaos, nonlinear parametric resonances, and hysteretic behavior, etc. All these nonlinear effects have been observed with time-symmetric, typically sinusoidal, modulations. Triangular modulations have also been considered in the context of systems with a swept parameter, in particular, in the study of “delayed bifurcations” observed when the control parameter is swept in the vicinity of a bifurcation to explore the properties of switching from an unstable to a stable state.

In these studies, emphasis was put on the fact that sweeping the control parameter induces shifts between static (i.e., in the absence of sweep) and dynamic (i.e., with a swept parameter) bifurcations. Up and down sweeps produce different shifts, and fluctuations were shown to play a key role in these shifts. In the present work, we use periodic modulations instead of single sweeps and consider similar effects in a different parameter range, more specifically faster sweep rates, and we pay special attention to the effect of the asymmetry of the modulation, which is shown to have a dramatic influence on the system response.

Experiments have been performed on a Nd3+:YVO4 diode-pumped laser, and the modulation is applied by changing the power delivered by the pumping laser diode. The modulation is applied by changing the power delivered by the pumping laser diode, such that the time evolution of the pump power is periodic and has a triangular shape with a
variable up–down ratio and a fixed frequency. In the experiments shown hereafter, the modulation period is $T=84 \, \mu s$. This value is chosen as much larger than the relaxation oscillation (RO) period and falls in the typical range for the switch-on times associated with the buildup of laser action. For the parameters used, i.e., a pumping rate $A$ varying from $A_{\text{min}}=0.68$ to $A_{\text{max}}=1.2$, the RO period is typically 6 $\mu s$ and the switch-on time, which is known to diverge as the threshold is approached, is approximately 30 $\mu s$.

The output power delivered by the laser is monitored in stationary conditions, typically 400 periods, so that the transients associated with the initial switch on are damped. The relative duration of the up and down slopes appears to strongly influence the laser output. The asymmetry of the laser driving may be characterized by the ratio $\alpha$ of the up-going part to the triangle period so that 0.01 and 0.99 correspond, respectively, to the negative and positive sawtooths as shown in Fig. 1(a). Drastically different responses are observed for these two values of $\alpha$. For $\alpha=0.99$, the sawtooth modulation produces large spikes near the end of the modulation period, while the laser always remains off for opposite modulation ($\alpha=0.01$). For symmetric triangles ($\alpha=0.50$), spikes are also observed but with an intensity significantly reduced (typically by a factor of 2) with respect to the case $\alpha=0.99$. Note that in all these experiments, the average value of the pump power remains the same, while the laser either delivers coherent radiation or not, depending on the shape of the modulation. Therefore only a suitably tailored modulation produces a directed laser output.

The spike amplitude is not simply proportional to the asymmetry factor, as could be expected from the results displayed in Fig. 1(a). In fact in the corresponding experimental conditions, the laser never starts for $\alpha$ less than 0.26, and the spike amplitude, which is maximum for $\alpha=0.80$, is slightly reduced for the full asymmetric sawtooth ($\alpha=0.99$) [see Fig. 2(a)]. Moreover, the spike amplitudes suffer from strong fluctuations, which originate mainly from the stochastic character of spontaneous emission. Fluctuations due to spontaneous emission are a necessary ingredient, since in the absence of spontaneous emission, the laser would stay forever in the stationary OFF state. Indeed, the initial conditions at the beginning of a modulation period are fixed by the radiation left over by the preceding pulse if the laser is fired during this period. If the modulation is slow enough that this radiation is less than the spontaneous emission level, the latter effect dominates, hence its stochastic character is transferred onto the pulse delivered by the laser. As usual, both amplitude and delay are affected by these initial conditions. As shown in Fig. 2(b), there are two regimes. At low symmetry parameter values ($0.25<\alpha<0.55$), spontaneous emission is the dominant triggering mechanism, and the fluctuations increase with the delay, as observed in a standard laser subjected to a square-modulated pump. On the contrary, at large values ($0.55<\alpha<0.99$), the fluctuations slowly decrease while the delay increases, and the effect of the spontaneous emission level on the delay becomes negligible, although it remains necessary to initialize the startup of the laser. In this peculiar regime, where the laser fires a large pulse associated with a large population depletion, the laser behavior is dominated by the interplay between population and field dynamics. For a given amplitude of pump modulation, the effect of the pump asymmetry decreases as the average value of the pump increases and disappears when this value becomes larger than the threshold.

These experimental results are compared with the predictions of a model for a class B laser including a noise contribution $\varepsilon(t)$ in the field equation to account for spontaneous emission. It reads as

$$
\frac{dE(t)}{dt} = E(t)[D(t) - 1] + \varepsilon(t),
$$

$$
\frac{dD(t)}{dt} = \gamma[A - D(t)[E^2(t) + 1]],
$$

where $E$, $D$, and $A$ are the field, the population inversion, and the pumping rate, respectively. $\gamma = \gamma/\kappa$ is
the population damping rate $\gamma$ in units of the field damping rate $\kappa$ and $t$ is the time in units of the field damping time $1/\kappa$. $\varepsilon(t)$ is a correlated noise (Gaussian-distributed Ornstein–Uhlenbeck process) of zero mean and correlations given by

$$\langle \varepsilon(t)\varepsilon(s) \rangle = \frac{Q}{\tau} \exp\left(-\frac{|t-s|}{\tau}\right). \tag{2}$$

In our simulations the correlation time $\tau=9\times10^{-2}$ is much smaller than 1, so that the noise is weakly correlated and can be considered as white noise. The values of $\kappa$ and $\gamma(\kappa=9.1\times10^7$ s$^{-1}, \gamma=3.66\times10^{-4}$) are those measured on our laser by fitting its switch-on dynamics on several models. The noise strength $Q=4.7\times10^{-9}$ has also been fitted to reproduce these dynamics. Numerical simulations have been performed using the procedure introduced by Fox$^{12}$ for the integration of stochastic differential equations. An excellent agreement is obtained with the experimental results as shown in Fig. 1(b). Figure 3 shows that the value of the minimum symmetry parameter $\alpha$ required for laser action is less than 0.20 in the simulations, whereas this value is 0.26 in experiments, but all the qualitative phenomena shown in Fig. 2 are recovered. Such numerical simulations allow us to compare the results obtained in the presence and in the absence of fluctuations, and consequently to identify the role played by the stochastic terms in the dynamics. They also give access to a variable such as $D$, which is not easily accessible in experiments. In particular, it is possible to evaluate the total energy transferred from the pump to the population, which writes $\int_0^T (D-D_0)\,dt$ where $D_0$ is the population difference at the beginning of a modulation period. Simulations show that with a negative sawtooth ($\alpha<0.5$), the population inversion growth is faster and the total energy transferred is larger than with a positive sawtooth ($\alpha>0.5$). Although a positive sawtooth leads to less energy deposited in the laser medium, the population difference may temporarily reach a larger value, making it possible to exceed the threshold in this case, while it does not with the reverse (negative) modulation. For instance, in the conditions of our experiments, the energy deposition in the active medium with $\alpha=0.01$ is 50% larger than with $\alpha=0.99$, but the maximum values of $D$ are 0.953 and 1.025, respectively, and the gain remains negative for the former value, while it may become positive for the latter. This population dynamics interferes with the stochastic dynamics due to the noise term that mimics spontaneous emission. We stress that, on the one hand, the laser stays on average well below threshold, and that, on the other hand, this modulation is too fast to be considered as simple adiabatic exploration of the above threshold region.

In conclusion, we have demonstrated a directed energy transfer that is reminiscent of the ratchet effect, but the roles of the fluctuations and of the asymmetric driving are reversed in the present work. The common point of both phenomena is that, here in the time domain, tailoring a suitable perturbation allows one to extract directed energy, in the form of coherent radiation of the laser, while a perturbation with opposite time symmetry leads to incoherent emission only. Although this is not rigorous, spontaneous emission in the lasers is often described by a source term proportional to $D$ in the field equation. This would be sufficient to trigger the laser oscillation, but would not account for the peculiar behavior described above for the fluctuations. Inversely, it is possible to introduce an additional stochastic term in the population equation to account for the technical fluctuations of the pump. This was shown not to produce any significantly new results. The case of strong noise and weak modulations, which corresponds more closely to the classical ratchet that will be investigated soon, requires new experimental developments to detect small amounts of coherent radiation from the large incoherent background.

B. Segard’s e-mail address is bernard.segard@univ-lille1.fr.

References


9. The delay is the time elapsed between the instant when the pump power equals the threshold and that when the pulse emission starts. It is normalized to the pump duration above threshold.