Nonlinear acrobatics of viscous filaments

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Résumé. De minces filaments de fluide visqueux jouent un rôle important dans de nombreux phénomènes naturels et procédés industriels. A cause de sa surface libre, un filament visqueux se comporte de façon non-linéaire de par sa nature même, avec notamment la possibilité d'états radicalement différents pour un seul et même jeu de paramètres externes. Nous étudions depuis quelque temps la riche dynamique 'multistable' d'un filament visqueux qui tombe verticalement sur une surface rigide, en conjuguant expériences de laboratoire et modélisation. Nous en présentons ici trois exemples : (1) Modes 'pendulaires' d'un filament qui s'enroule sur lui-même ; (2) Génération d'ondes spirales par l'instabilité d'enroulement ; et (3) Etats multiples d'un filament de faible viscosité.

Abstract. Thin filaments of viscous fluid play an important role in many natural phenomena and industrial processes. Because of its free surface, a viscous filament behaves in an inherently nonlinear fashion, and can exhibit radically different states for a given set of external control parameters. Using laboratory experiences and theoretical modeling, we have studied this 'multistable' dynamics in the particular case of a viscous filament falling onto a rigid surface. Here we present three examples : (1) 'pendular' modes of a coiling filament ; (2) generation of spiral waves by the coiling instability ; and (3) multiple states of a low-viscosity filament.

1 Introduction

Thin filaments of viscous fluid play an important role in a variety of natural and industrial contexts, from volcanic eruptions to the manufacture of polymer products and non-woven materials. In most of these applications, the filament responds to the different forces acting on it by adopting a complex three-dimensional shape that is not known *a priori*, and which must be determined either experimentally or numerically. Typically, the shape of the filament's axis is very far from any of the simple shapes (straight line, circular arc, etc.) for which the governing equations can be solved analytically. Consequently, the behavior of viscous filaments is inherently nonlinear, even if the rheology of the fluid in question is perfectly linear (Newtonian). One of the primary manifestations of this nonlinearity is a fundamental nonuniqueness of behavior, whereby a filament can exhibit multiple states with very different shapes for a given set of values of the control parameters that define the experimental situation.

Perhaps the best-known example of this sort of nonlinearity is the phenomenon of 'liquid rope coiling' [1], wherein a thin stream of viscous fluid (e.g., honey) falling onto a surface winds itself into a whirling 'corkscrew'. Fig. 1 shows the setup used in most experimental studies of this phenomenon, in which fluid with density ρ, viscosity ν and surface tension coefficient γ is ejected at a volumetric rate Q from a hole of diameter d and then falls a distance H onto a solid surface. For sufficiently large fall heights, the filament exhibits a two-part structure comprising a helical 'coil' and a long vertical 'tail'. The principal parameter of interest is the angular coiling frequency Ω.

During the past few years, we have studied liquid rope coiling using a combination of experimental, analytical, and numerical methods, and have found several new and surprising types of nonlinear behavior. In the rest of this article we shall present three of these : pendulum modes of highly viscous filaments ; generation of spiral waves by coiling ; and multiple states of low-viscosity filaments.

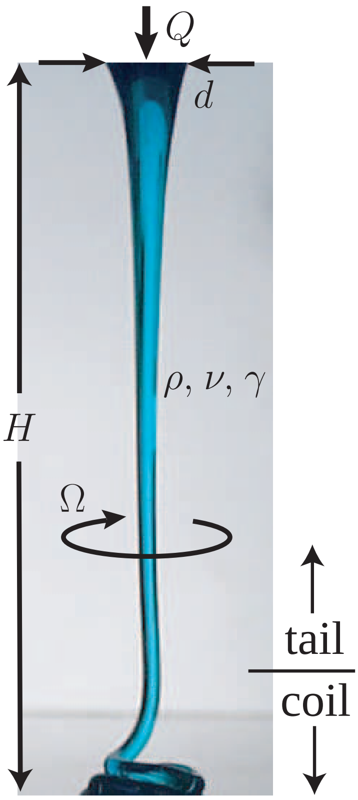

Fig. 1. Steady coiling of a 'rope' of viscous corn syrup (photograph by N. Ribe.) Fluid with density ρ, viscosity ν and surface tension coefficient γ is injected at volumetric rate Q through a hole of diameter d and falls a distance H onto a plate. The angular coiling frequency is Ω.

2 Pendulum modes of highly viscous filaments

To establish the context in which these modes occur, we need first to review briefly the different dynamical regimes of liquid rope coiling. These regimes are now fairly well understood thanks to more than 50 years of study using a variety of experimental [1,2,3,4,5,6,7,8,9,10,11] and theoretical [4,9,10,12,13,14] approaches. The results are summarized in Fig. 2a in the form of a plot of coiling frequency vs. height measured experimentally (symbols) and calculated numerically (solid curve) for $\rho = 0.97$ g cm^{-3}, $\gamma = 21.5$ dyne cm^{-1}, $\nu = 1000$ S, $d = 0.068$ cm and $Q = 0.00215$ cm^3 s^{-1} the parameters of one of the laboratory experiments in [9]. Four different regimes are seen in succession as the fall height H increases, depending on the relative magnitudes of the viscous, gravitational, and inertial forces acting on the filament. For small heights $H < 0.8$ cm, both gravity and inertia are negligible, and coiling occurs in a viscous (V) regime with a frequency that decreases with height. This regime is similar to the coiling of toothpaste squeezed from a tube against an impermeable surface. A second regime is seen in the range $H = 0.8 - 7$ cm. Here the inertia of the filament is still negligible, but gravity is now large enough to balance the viscous forces in the coil, leading to a gravitational (G) regime in which Ω increases with H. A third, inertial (I) regime appears at large fall heights $H > 15$ cm; here, the viscous forces in the coil are balanced by inertia, and gravity is negligible.

The most complicated and interesting behavior, however, is seen in the 'pendular' regime that occurs for intermediate fall heights $H = 7 - 15$ cm. In this range, the curve of coiling frequency vs. fall height is multivalued, implying that more than one coiling state can exist at a given fall height. In this regime, the 'tail' portion of the filament (Fig. 1) behaves as a whirling 'liquid pendulum' in which centrifugal acceleration is balanced by a combination of gravity and the viscous forces associated with extensional deformation. Ribe et el. [9] showed that such an object has an infinite series of eigenmodes with frequencies Ω_n proportional to the classical pendulum frequency $(g/H)^{1/2}$. If one of these eigenfrequencies happens to be close to the (G-regime) frequency set by the 'coil' portion of the filament, then the tail executes large-amplitude resonant oscillations. The first three liquid-pendulum eigenfrequencies $\Omega_1 - \Omega_3$ are indicated by the lines with slope $-1/2$ in Fig. 2, and are seen to correspond precisely to the 'bumps' on the curve of frequency vs. height.

A final interesting aspect of the pendular regime is that the dashed portions of the calculated frequency-height curve in Fig. 2 are unstable to small perturbations [10]. This is in agreement with the fact that steady coiling states (symbols in Fig. 2) are never observed along these portions of the curve in the laboratory.

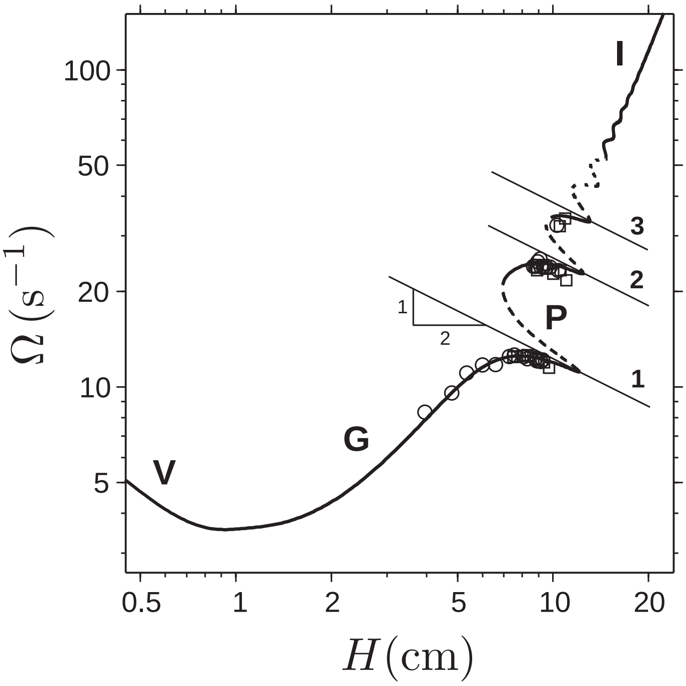

Fig. 2. Coiling frequency vs. fall height measured experimentally (symbols) and calculated numerically using the method of [14] (solid-dashed line) for silicone oil ($\rho = 0.97$ g cm^{-3}, $\gamma = 21.5$ dyne cm^{-1}, $\nu = 1000$ S) with $d = 0.068$ cm and $Q = 0.00215$ cm^3 s^{-1}. Dashed portions of the solid curve are unstable to small perturbations [10]. The labels indicate the four coiling regimes : viscous (V), gravitational (G), pendular (P), and inertial (I). The three lines with slope $-1/2$ indicate the first three 'liquid pendulum' eigenfrequencies.

3 Spiral waves generated by liquid rope coiling

In liquid rope coiling in general, each new 'loop' of the filament laid down lies exactly on top of the previous one. In a limited region of the parameter space, however, this is not the case : each new loop is systematically displaced relative to the previous one, in such a way that the center of coiling executes a slow retrograde progression with a frequency roughly one-fourth that of the coiling itself. By some mechanism not yet understood, the relative displacement of successive loops causes air bubbles to be trapped between them. These air bubbles are then expelled radially in the thin layer of fluid spreading away from the coiled loops over the impermeable surface, creating bubble patterns in the form of spiral waves.

Several examples of such waves are shown in Fig. 3. The top panel shows the typical appearance of the spirals viewed from above, and illustrates the fact that they always have five distinct branches. Moreover, if the sense of rotation of the coiling changes spontaneously, the curvature of the branches of the spiral follows suit, changing sign after a short lag time (compare images (a) and (b) in the top panel of Fig. 3). The lower panel of Fig. 3 shows that the spiral pattern depends critically on relatively small changes in the fall height. Habibi et al. [15] proposed a simple kinematic model involving two frequencies and two radii (of the coiling and the precession) that predicts the observed spiral patterns quite well.

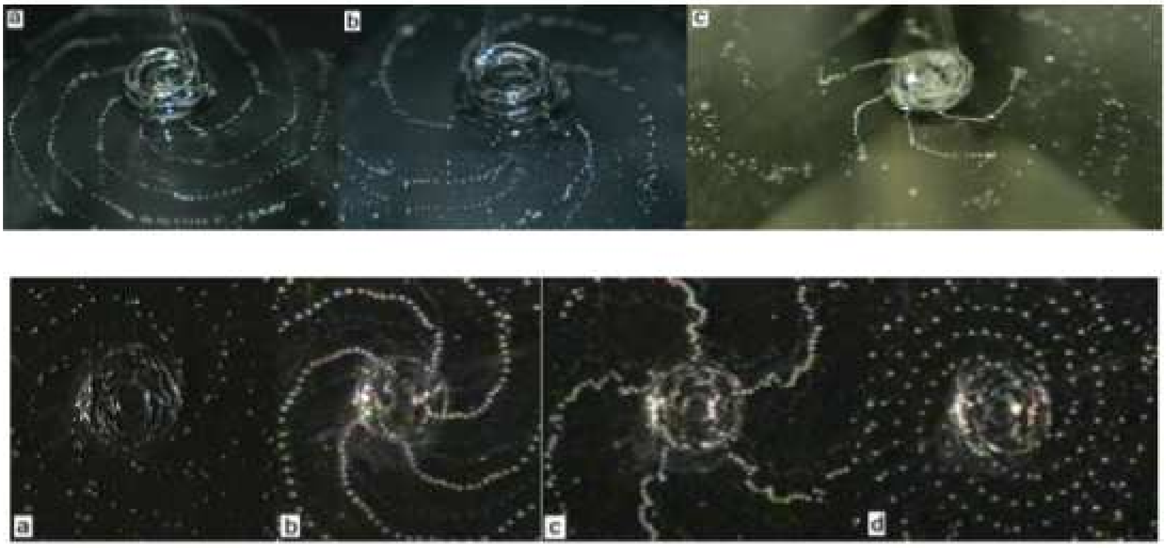

Fig. 3. Examples of spiral bubble waves generated by coiling of a filament of silicone oil with $\nu = 300$ S. Top : photographs taken obliquely from above. The diameter of the fluid pile in the center is about 1 cm. Bottom : photographs from below illustrating the variation of the spiral pattern with fall height in an experiment with $d = 1.6$ mm and $Q = 0.137$ cm^3 s^{-1}. (a) $H = 3$ cm, (b) 3.5 cm, (c) 3.7 cm, (d) 4.0 cm.

The next step in our investigation of this problem, still in progress, will be to go beyond kinematics to understand the dynamical basis of spiral wave formation. In this context, a key observation we need to understand is that spiral waves form only when the fall height is just below the value for the onset of the pendular (P) regime (Fig. 2). Another important question is the nature of the bifurcation from 'normal' coiling (with precisely superposed loops) to the 'coiling with precession' that gives rise to spiral waves. A preliminary analysis (unpublished) suggests that the bifurcation may be subcritical, but this requires further confirmation. Additional questions of interest include the following : Why is the precession frequency always about one-fourth that of the coiling ? Why is the precession retrograde, i.e. in the direction opposite to the coiling ? What is the precise mechanism by which bubbles are trapped between the coiled loops ?

4 Multiple states of low-viscosity filaments

Our final example of the nonlinear behavior of viscous filaments is the recent discovery [16] of three radically different states in a filament of low-viscosity fluid falling onto an impermeable surface. Fig. 4 shows these states as observed in an experiment using silicone oil ($\rho = 0.97$ g cm^{-3}, $\gamma = 21.5$ dyne cm^{-1}, $\nu = 5$ S) with $d = 2.6$ mm, $Q = 0.19$ cm^3 s^{-1}, and $H = 14$ cm. The first state (Fig. 4a) is simple stagnation flow, in which the filament retains its axial symmetry while thickening downward towards the surface. The second state (Fig. 4b) is steady coiling in the inertial (I) regime, similar to that seen previously using fluids with much higher viscosities. The most interesting state, however, is the third (Fig. 4c-d), in which the filament folds in (approximately) a vertical plane while that plane itself simultaneously precesses. While the folding frequency is only about 10% less than the frequency of the coiling shown in Fig. 4b, the precession frequency is smaller than both by a factor ≈ 50. The images in Fig. 4c and Fig. 4d are separated in time by one-half the precession period, and illustrate how the appearance of the