Auto-résonance de l'instabilité Raman stimulée due à une non-linéarité d'origine cinétique dans un plasma inhomogène

Thomas Chapman[1], Stefan Hüller[1], & Paul-Edouard Masson-Laborde[2] & Wojciech Rozmus[3]

[1] Centre de Physique Théorique, CNRS, École Polytechnique, Palaiseau, France
[2] CEA, DAM, DIF, F-91297 Arpajon, France
[3] University of Alberta, Edmonton, Alberta, Canada
chapman@cpht.polytechnique.fr

Résumé. Nous présentons de nouvelles solutions du système à trois ondes couplées décrivant les instabilités paramétriques de diffusion. L'étude analytique et numérique est focalisée sur l'influence de l'effet d'auto-résonance dans un plasma inhomogène, dans le cas où la non-linéarité de l'onde de plasma est dominée par l'interaction onde-électrons (non-linéarité de type cinétique). Dans le contexte de la fusion inertielle par laser, on montre que l'onde plasma électronique peut être verrouillée en phase avec la force pondéromotrice associée au battement entre l'onde laser et l'onde de la lumière rétro-diffusée. Cette solution verrouillée en phase peut donner lieu à une importante croissance séculaire de l'onde plasma, différente de celle habituellement observée. On montre que la croissance de cette onde induit une rupture du verrouillage de phase, qui met fin au développement de l'instabilité.

Abstract. New solutions to the coupled three-wave equations describing parametric scattering instabilities are presented. This analytical and numerical study investigates the impact of autoresonance on stimulated Raman scattering in an inhomogenous plasma, where the dominant plasma wave nonlinearity is due to wave-particle interactions (a kinetic-type nonlinearity). Under conditions in the plasma relevant to laser fusion, it is shown that the electron plasma wave may become phase-locked to the beating between a laser pump wave and a back-scattered light wave. This phase-locked solution may grow to a significant amplitude. It is shown that the growth of the electron plasma wave will lead to a break-down of the phase-locking and hence its maximum attainable amplitude is inherently limited.

1 Introduction

Stimulated Raman Scattering (SRS) in warm plasmas is a parametrically unstable resonant three-wave interaction, where laser light (the pump wave) scatters off an electron plasma wave (the Langmuir wave), resulting in a second electromagnetic wave (the scattered wave). The understanding of the SRS process is of central importance to the realisation of laser thermonuclear fusion. Through back-scattering of the laser and fast-electron generation, SRS reduces the efficiency of the laser light absorption that heats the plasma corona. This in turn inhibits the necessary ablation and compression processes and may eventually inhibit the ignition of the thermonuclear deuterium-tritium fuel.

In the presence of a nonlinear perturbation such as wave-profile steepening or kinetic effects, the Langmuir wave can be considered as a nonlinear oscillator coupled to the electromagnetic waves [1]. The frequency of a nonlinear oscillator varies with the amplitude of oscillation. Starting from rest, if the system is driven at its linear (constant) frequency, then the amplitude of oscillation will initially increase. However, the oscillator will quickly dephase from the drive and the amplitude will subsequently beat back to a small value. If instead the system is driven at a frequency that also varies, the oscillator may, under certain conditions, phase-lock to the drive and automatically stay resonant, allowing the amplitude to grow well beyond the response to a fixed-frequency drive.

Autoresonance arises in many systems such as driven pendulums [2], electron beams in accelerators [3,4] and superconducting Josephson junctions [5]. Yaakobi *et al.* [1] investigated the phenomenon of autoresonance in the SRS process in a physical regime where the dominant nonlinear frequency shift (NFS) of the oscillation (in this case, in the Langmuir wave equation) arose from thermal and relativistic effects in the plasma. This fluid-type NFS was proportional to the squared Langmuir wave amplitude.

In the following work, we investigate autoresonance in the SRS process in the physical regime where the dominant NFS arises from kinetic effects. This kinetic-type NFS is proportional to the square root of the Langmuir wave amplitude [8].

2 The three-wave equations with a kinetic-type nonlinearity

We consider the case where the incident electromagnetic pump wave (forwards-propagating) couples to the Langmuir wave (forwards-propagating) and a back-scattered electromagnetic wave (backwards-propagating) in a plasma with a density that increases linearly in the forwards direction. The SRS process can be described by the following system of quadratically-coupled envelope equations (see e.g. Ref. [6]), relevant to a one-dimensional, weakly non-uniform, stationary, underdense, thermal plasma :

$$\left(\frac{\partial}{\partial t} + c_0\frac{\partial}{\partial z}\right) a_0 = -\frac{\omega_p^2}{2\omega_0} a_1 a_L, \tag{1}$$

$$\left(\frac{\partial}{\partial t} - |c_1|\frac{\partial}{\partial z}\right) a_1 = \frac{\omega_p^2}{2\omega_1} a_0 a_L^*, \tag{2}$$

$$\left(\frac{\partial}{\partial t} + c_L\frac{\partial}{\partial z} + i\beta|a_L|^{1/2} - ic_L\kappa' z\right) a_L = \gamma\omega_0 a_0 a_1{}^*. \tag{3}$$

We describe each of the three waves in terms of a slowly-varying complex envelope and a quickly-varying phase. The envelopes a_0 and a_1 describe the pump and scattered waves respectively and the Langmuir wave is described by $a_L = \delta n_e/n_0$ where n_0 is the plasma density before the formation of the Langmuir wave and δn_e is the perturbation of this density.

We assume the three-wave resonance condition $\omega_0 = \omega_1 + \omega_L$ for the frequencies, while the wave vectors $k_{0,1,L}$ are slowly varying functions of space z. The SRS process is relevant in the regime where all frequencies are greater than $\omega_p/2$, where the plasma frequency $\omega_p^2 = \omega_p^2(z) = n_0 e^2/\epsilon_0 m_e$, for which ϵ_0 is the permittivity of free space, e is the charge of the electron and m_e is the electron mass. The wave vectors are mismatched according to $k_0 - k_1 - k_L = \kappa' z$, where κ' parameterises the spatial inhomogeneity (derived by Liu *et al.* [7]), and act to detune the three waves from resonance either side of $z = 0$. The vacuum speed of light is given by c while the group velocities of the three waves are $c_{0,1,L}$. The coupling strength of Langmuir wave to the pump and scattered waves is given by the parameter $\gamma = (k_L^2 c^2/2\omega_L\omega_0)(I_F/n_c mc^3)$ where I_F is the incident pump wave energy flux and n_c is the density up to which the pump wave can penetrate (the critical density).

The frequency shift of the Langmuir wave $-ic_L\kappa' z$ in equation (3) is due to the mismatch in the wave vectors, with the spatial resonance occurring at $z = 0$. This study focuses on the kinetic NFS $i\beta|a_L|^{1/2}$ in equation (3) and its potential to balance the frequency shift due to the wave vector mismatch over a significant distance beyond the resonance point. The strength β of the kinetic NFS is assumed constant in all simulations.

3 Analysis using a prescribed ponderomotive drive

We begin this investigation of spatial autoresonance by initially reducing the system of three coupled equations to a single equation describing the Langmuir wave. We prescribe the amplitudes of the electromagnetic waves on the RHS of equation (3) and replace $\gamma\omega_0 a_0 a_1{}^*$ with a prescribed ponderomotive drive P. This allows the investigation of spatial autoresonance to be uncomplicated by effects due to the depletion of the pump wave or growth of the scattered wave. Furthermore, the Langmuir wave is driven by a single frequency meaning SRS may occur only at the resonance point.

For convenience of analysis, we define the characteristics $t = s - s_*$ and $z = c_L s$, where s is a real variable chosen so that $s = 0$ at $z = 0$ and s_* is a constant. The characteristics are straight lines in the (z, t)-plane, intercepting the z-axis at s_* and bound by z_L and z_R (the left and right boundaries of

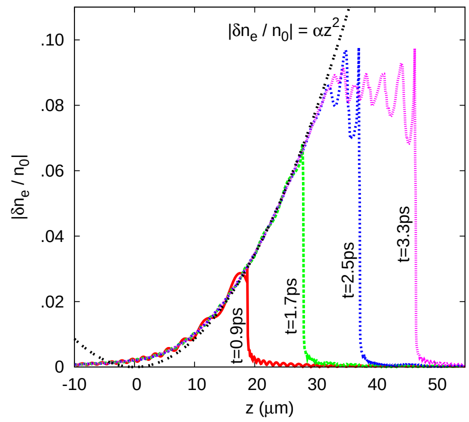

Fig. 1. (Colour online) Solutions to the prescribed ponderomotive drive Langmuir wave equation (4) showing the growth and propagation of the Langmuir wave through a positive plasma density gradient at different times, t=0.9, 1.7, 2.5, 3.3 ps. The equation $|\delta n_e/n_0| = \alpha z^2$ where $\alpha = (c_L\kappa'/\beta)^2 = 8\times10^{-5}$ μm^{-2} describes the growth of the leading edge of the Langmuir wave, based on the cancellation of the kinetic nonlinear frequency shift and the frequency shift due to the wave vector mismatch.

the plasma respectively). Using the total derivative of a_L, we may rewrite equation (3) as the ordinary differential equation

$$\frac{da_L}{ds} + i(\beta|a_L|^{1/2} - c_L^2\kappa' s)a_L = P(s), \qquad (4)$$

which is readily solved along these characteristics. Equation (4) must then be solved for a range of values of s_* with the initial condition $a_L(z = z_L) = 0$ in order for the space-time dependence of the Langmuir wave equation to be recovered. For $t < 0$, $P(s) = 0$ and a_L is undriven. At $t = 0$, we switch on the prescribed ponderomotive drive. For $t \geq 0$, $P(s)$ is assigned a fixed value, uniform throughout the plasma hence $P(s) = \mu H(s - s_*)$, where μ is a constant and H is the Heaviside step function.

Following the results of Yaakobi *et al.* [1], we look for solutions to equation (4) characterised by the autoresonant growth of a_L to high amplitude, where the phase $\phi_L = \arg(a_L)$ is constant over a region in space (i.e. where a_L is phase locked to P). We also expect solutions characterised by ϕ_L changing rapidly and a_L oscillating around a constant value. In Fig. 1, we present a solution to equation (4), showing the propagation and evolution of a_L through space and time. For consistency, the values of β and P were chosen to be in agreement with the parameters used in section 4 (P is in this case real and negative due to the locked phases $\phi_0 = \arg(a_0) \approx 0$ and $\phi_1 = \arg(a_1) \approx -\pi$, calculated in section 4). The solution is autoresonant with a growing wave front closely following the curve

$$|a_L| = (c_L\kappa'/\beta)^2 z^2 \quad \text{or} \quad \beta|a_L|^{1/2} = c_L^2\kappa' s \qquad (5)$$

for $z, s > 0$, indicating a cancellation of the kinetic NFS and the frequency shift due to the plasma inhomogeneity. The phase $\phi_L \approx \pi/2$ throughout the region behind the wave front. Behind the wave front, a_L is steady in space and time but for small fluctuations.

The solutions presented by Yaakobi *et al.* [1] for the growth of the Langmuir wave in a regime where the dominant NFS is of a fluid type displayed a pronounced threshold phenomenon. In this case, the Langmuir wave was observed to dephase quickly from the drive when driven below threshold. When driven above this threshold, however, the phase-locking of the Langmuir wave to the drive continued indefinitely, allowing the Langmuir wave to grow arbitrarily high if pump depletion effects were not introduced. This behaviour was explained by Fajans and Frièdland [2] by drawing analogy to a driven pendulum and considering the formation of pseudo-potential wells that act to trap the phase of the pendulum, leading to small oscillations of ϕ_L around a constant value. A similar analysis of theses pseudo-potential wells where a kinetic NFS replaces the fluid NFS may be performed. By substituting $a_L = |a_L|\exp(i\phi_L)$ into equation

(4), we may write

$$\frac{dI}{ds} = -2I^{1/2}|P|\cos(\phi_L), \tag{6}$$

$$\frac{d\phi_L}{ds} = c_L^2\kappa' s - \beta I^{1/4} + I^{-1/2}|P|\sin(\phi_L), \tag{7}$$

where $I = a_L^2$ is the action of the system. From solutions to equation (4), we observe the phase to be locked at $\phi_L \approx \pi/2$ as the system passes through resonance at $s = 0$. For the system to remain autoresonant and for the drive P to be effectively coupled to I, the phase must remain locked at this value. We separate I into the slowly-varying average action I_0 and a perturbation Δ, where $I = I_0 - \Delta$. Since ϕ_L is constant during autoresonance, we may set its derivative in equation (7) to zero and using the average action write

$$0 \approx c_L^2\kappa' s - \beta I_0^{1/4} + I_0^{-1/2}|P|. \tag{8}$$

Differentiating this expression and solving for I_0, we find

$$\frac{dI_0}{ds} = \frac{c_L^2\kappa'}{M}, \tag{9}$$

where the slowly-varying parameter $M = M(s)$ is

$$M = \frac{|P|}{2I_0^{3/2}} + \frac{\beta}{4I_0^{3/4}}. \tag{10}$$

We now expand equations (6) and (7) around the instantaneous value of the average action I_0 by substituting equation (9) into equation (6) and equation (8) into equation (7). To lowest order, we find

$$\frac{d\Delta}{ds} = 2I_0^{1/2}|P|\cos(\phi_L) + \frac{c_L^2\kappa'}{M}, \tag{11}$$

$$\frac{d\phi_L}{ds} = \Delta M. \tag{12}$$

Equations (11) and (12) form a Hamiltonian system $\mathcal{H} = \mathcal{H}(\Delta, \phi_L, s)$ where

$$\mathcal{H} = \frac{M\Delta^2}{2} - \left(2I_0^{1/2}|P|\sin(\phi_L) + \frac{c_L^2\kappa'}{M}\phi_L\right) = \frac{1}{2M}\left(\frac{d\phi_L}{ds}\right)^2 + V(\phi_L). \tag{13}$$

We may interpret equation (13) as a Hamiltonian governing the behaviour of a pseudoparticle at $s \approx 0$ that describes the phase ϕ_L of the Langmuir wave. This pseudoparticle has a slowly-varying effective mass M and travels through a potential V that varies with ϕ_L. The potential $V = V_{lin} + V_{osc}$ is a sum of two terms : a linear term $V_{lin} = -(c_L^2\kappa'/M)\phi_L$ and a series of potential wells $V_{osc} = -2I_0^{1/2}|P|\sin(\phi_L)$. Under the condition that M varies slowly, these wells may be significant and can act to trap the pseudoparticle provided that

$$\left|\frac{dV_{osc}}{d\phi_L}\right| > \left|\frac{dV_{lin}}{d\phi_L}\right|, \tag{14}$$

for a range of values of ϕ_L within the period of V_{osc}. In this case, the slope of V_{lin} is small enough that the oscillation of V_{osc} causes an overall oscillation in the sign of the gradient of V. This will be true while

$$2I_0^{1/2}|P| > c_L^2\kappa'/M. \tag{15}$$

Since we have assumed that the system is initially autoresonant, we begin with the pseudoparticle trapped in V and ϕ_L performing small oscillations around $\pi/2$. Initially, the average action I_0 grows according to $I_0 \approx |a_L|^2 \propto s^4$. However, as I_0 increases, we see from equation (10) that M decreases quickly and that the RHS of equation (15) increases faster than the LHS of equation (15). This results in a weakening of the pseudoparticle trapping and the eventual loss of autoresonance, shown in Fig. 2.

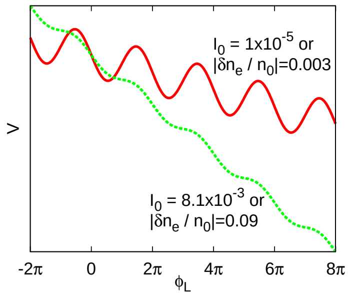

Fig. 2. (Colour online) The potential $V = -(c_L^2\kappa'/M)\phi_L - 2I_0^{1/2}|P|\sin(\phi_L)$ shown at $I_0 = 1\times10^{-5}$ (solid red line) and $I_0 = 8.1\times10^{-3}$ (dashed green line) corresponding to $z\approx 0$ μm and $z\approx 35$ μm in Fig. 1 respectively. All parameters used to calculate V other than I_0 are identical in the two cases. The potential wells are capable of trapping the phase of the Langmuir wave ϕ_L while $2I_0^{1/2}|P| > c_L^2\kappa'/M$. The potential wells become less significant and eventually disappear as the average action I_0 increases, ending the autoresonant region in space. For clarity, V has been increased by a factor of 100 for $I_0 = 1\times10^{-5}$ (solid red line).

The sharp threshold behaviour of autoresonance arising from a fluid NFS found by Yaakobi *et al.* [1] is not observed in the kinetic case. Regardless of the parameters chosen, autoresonance where the dominant NFS is kinetic will always eventually be lost as s (or z), and consequently I_0, increases. In Fig. 1, we observe this breakdown of phase-locking and the resulting plateau of the amplitude at $|\delta n_e/n_0| = 0.09$. From equation (15), it is clear that this maximum amplitude increases with the magnitude of both β and P. However, due to the β^{-2} dependence of the Langmuir wave amplitude in equation (5), the wave front will grow more slowly in space as β is increased and require a longer distance to reach a plateau. Equation (15) may be solved for I_0, and using unchanged parameters, we find a predicted maximum amplitude $|\delta n_e/n_0| = 0.06$. This is lower than the actual maximum reached in the solutions shown in Fig. 1 since the growth of ϕ_L will still be slow for a short distance in space after the initial loss of the potential wells, resulting in a continued increase in $|\delta n_e/n_0|$.

4 Three-wave coupling code with a positive density gradient

Using a finite difference method, we numerically solve the space-time dependent system of equations (1-3) between the boundaries z_L and z_R. At z_R, we seed the system by injecting a back-scattered wave from a noise source (here, we use a Langevin equation) with a broad frequency spectrum, allowing SRS to occur at all points in the plasma. At z_L, a pump wave is injected after ensuring that the injected back-scattered wave has propagated throughout the plasma. To better approximate the physical situation, we include a nonlinear Landau damping term $\nu = 0.03\omega_p[1+\beta|a_{Lmax}|^{1/2}(t/\tau)]^{-1}$ in equation (3), where τ is the time for resonant trapped electrons to bounce across the potential well of the wave and a_{Lmax} is the maximum local value of the Langmuir wave which propagates at the phase velocity $c_{ph}\equiv\omega_L/k_L$.

In Fig. 3 we present solutions to equations (1-3), assuming a peak pump wave intensity $I_F = 5\times10^{15}$ W/cm^2 with wavelength $\lambda_0 = 2\pi c/\omega_0 = 351$ nm, and an injected back-scattered wave at an average thermal level corresponding to an intensity of approximately 5×10^4 W/cm^2. The inhomogeneity parameter $\kappa' = 4.4\times10^7$ cm^{-2}. The wave vector of the Langmuir wave k_L and the Debye length λ_{De} are such that the plasma parameter $k_L\lambda_{De}(z=0)\simeq 0.34$ is in a typically "kinetic" regime.

In Fig. 3(a), $a_L = \delta n_e/n_0$ is shown as a function of z at a range of times showing the evolution of the Langmuir wave. We find that the leading edge of the Langmuir wave front follows the relation found using a prescribed ponderomotive drive given in equation (5). In Fig. 3(b), the phase is observed to be locked at $\phi_L\approx\pi/2$ over a distance that increases with the same speed that the wave front of a_L propagates. The significant deviation of the phase from this value occurs simultaneously to the collapse of the steady form of the Langmuir wave front. In Fig. 3(c), the effective cancellation between the kinetic NFS and the

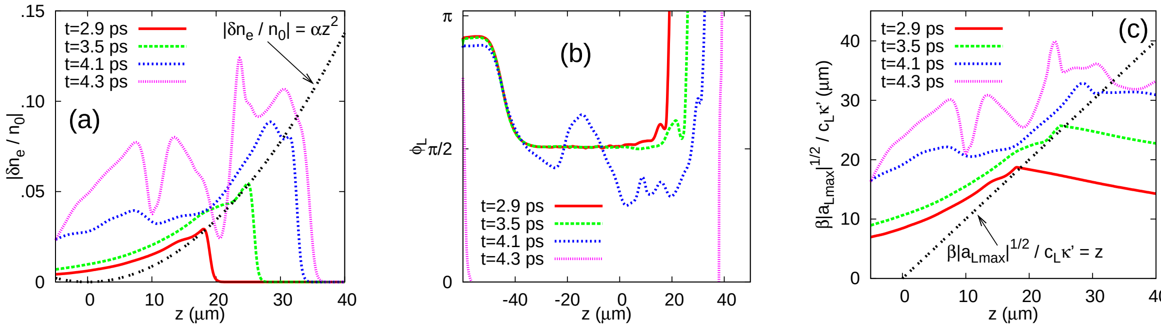

Fig. 3. (Colour online) Solutions to the three-wave equations (1-3). (a) the growth and propagation of the Langmuir wave is shown at different times, t=2.9, 3.5, 4.1, 4.3 ps. The equation $|\delta n_e/n_0| = \alpha z^2$ where $\alpha = (c_L\kappa'/\beta)^2 = 8 \times 10^{-5}$ μm^{-2} describes the growth of the leading edge of the Langmuir wave, based on the cancellation of the kinetic nonlinear frequency shift and the frequency shift due to the wave vector mismatch. (b) the phase ϕ_L associated with the Langmuir wave. (c) the magnitude of the kinetic NFS term in the wave equation of the Langmuir wave (normalised to the term describing the frequency shift due to the wave vector mismatch)

shift due to the spatial inhomogeneity may be seen. The cancellation is total only for a short distance in space at the leading edge of the Langmuir wave growth.

5 Conclusion

Under conditions relevant to laser fusion experiments, the autoresonant SRS-driven Langmuir wave obtained by solving the three-wave equations has a slowly-changing form with a wave front that grows according to $|\delta n_e/n_0| = \alpha z^2$, where $\alpha = (c_L\kappa'/\beta)^2$. When the pump wave passes through a positive density gradient, the autoresonant growth will begin at the three-wave resonance point and continue until the growing amplitude of the Langmuir wave causes a break-down in the phase-locking of the Langmuir wave to the pump and back-scattered waves. This loss of autoresonance is inevitable and is a property of the form of the kinetic nonlinear frequency shift. Due to the inherent limit on the maximum Langmuir wave amplitude that can be reached in this way, the length in space over which autoresonance may occur is also limited.

Références

1. O. Yaakobi, L. Friedland, R. R. Lindberg, A. E. Charman, G. Penn & J. S. Wurtele, Spatially autoresonant stimulated Raman scattering in nonuniform plasmas, *Physics of Plasmas*, **15**, 032105 (2008)
2. J. Fajans & L. Friedland, Autoresonant (non stationary) excitation of a pendulum, Plutinos, plasmas and other nonlinear oscillators, *American Journal of Physics*, **69**, 1096 (2001).
3. E. M. McMillan, The Synchrotron — A Proposed High Energy Particle Accelerator, *Physical Review E*, **68**, 143 (1945).
4. V. I. Veksler, *J. Phys.* (U.S.S.R.) **9**, 153 (1945).
5. O. Naaman, J. Aumentado, L. Friedland, J. S. Wurtele & I. Siddiqi, Phase-locking transition in a chirped superconducting Josephson resonator, *Physical Review Letters*, **101**, 117005 (2008).
6. D. Forslund, J. Kindel & E. Lindman, Theory of stimulated scattering processes in laser-irradiated plasmas, *Physics of Fluids*, **18**, 1002 (1975).
7. C. S. Liu, M. N. Rosenbluth & R. B. White, Parametric scattering instabilities in inhomogenous plasmas, *Physical Review Letters*, **31**, 697 (1973).
8. G. J. Morales & T. M. O'Neil, Nonlinear frequency shift of an electron plasma wave, *Physical Review Letters*, **28**, 417 (1972).