

Chaotic dynamics of a convection roll in a highly confined, vertical, differentially heated fluid layer

Zhenlan GAO^{1,2,3}, Bérengère Podvin¹, Anne Sergent^{1,2}, & Shihe Xin⁴

¹ CNRS, LIMSI, UPR3251, BP 133, 91403, Orsay Cedex, France

² Université Pierre et Marie Curie - Paris 06, 4 Place Jussieu, 75252 Paris, Cedex 05, France

³ Arts et Métiers ParisTech, 2 Boulevard du Ronceray, 49035 Angers Cedex 01, France

⁴ CETHIL, INSA de Lyon, 69621 Villeurbanne Cedex, France

`gao@limsi.fr`

The air flow between two differentially heated, vertical plates is characterised by cat's eye-like convection rolls when the Rayleigh number (Ra) is above a critical value. These convection rolls are found to be connected by oblique vorticity braids in the case of a transversely confined domain [1,2]. In this work we focus on the dynamics of a single convection roll by considering a small periodic domain, using direct numerical simulation (DNS) [2]. Via a Hopf bifurcation, the roll and braids grow and shrink alternatively and periodically [1]. As Ra increases, the flow becomes temporally chaotic through a period-doubling cascade [3,4], which is a new result, as chaos usually occurs through quasi-periodicity in laterally heated cavities [5]. The largest Lyapunov exponent of the flow [6] is found positive. The bifurcation diagram displays periodic windows as well as interior crises. The Feigenbaum constant based on the first few bifurcations is close to the theoretical value [4]. As Ra further increases, intermittency appears as the roll randomly switches between two vertical positions distant half the wavelength of the coherent structure, which is seen as an "attractor-merging" crisis [7]. The jump of the roll between two locations suggests the existence of a heteroclinic connection between two chaotic attractors, which form a $O(2) \times O(2)$ invariant torus. A critical crisis exponent [7] is computed to characterize the mean time between the switches.

In the spirit of [8], we derive a low-order model for the time evolution of the three principal spatial Fourier modes and show that some key features of the flow dynamics are correctly captured. The model successfully predicts the limit cycles which are close to the ones observed in DNS. The addition of a periodic perturbation to account for the effect of higher-order modes leads to a modulation of the cycles, which is reminiscent of the chaotic regime. Finally, we show that the presence of random noise in the system can generate strong excursions in phase space which mimic the roll shift observed in the simulation.

Références

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