Kinetic description

- VLASOV equation: dynamics of the distribution function $f(\mathbf{x}, \mathbf{v}, t)$ of particles in phase space

$$\frac{\partial f}{\partial t} = -\mathbf{v} \cdot \nabla f - \frac{q}{m} (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \frac{\partial f}{\partial \mathbf{v}} = \{f, \mathcal{H}\}$$

along with MAXWELL's equations for $\mathbf{E}(\mathbf{x}, t)$ and $\mathbf{B}(\mathbf{x}, t)$

- Hamiltonian structure: alternating bracket $\{\cdot, \cdot\}$ which is bilinear and satisfies the JACOBI identity

$$\{F, \{G, H\}\} + \{H, \{F, G\}\} + \{G, \{H, F\}\} = 0$$

Centre de Physique Théorique

Reductions

Aix*Marseille université

Fluid description

- dynamics in configurations space: $\mathbf{x}$ instead of $(\mathbf{x}, \mathbf{v})$
- infinite number of field variables (density, velocity, pressure, ...)
  $\Rightarrow$ need to close these equations

Objective: obtain closed fluid models that preserve the Hamiltonian structure of the kinetic equations

Framework

- 1D Vlasov-Ampère: $\partial_t f = -v\partial_x f + E\partial_v f$ and $\partial_t E = 4\pi \int vf \, \mathrm{d}v$
- 4+1 field model: density $\rho(x,t)$, velocity $u(x,t)$, pressure $P(x,t)$, heat flux $q(x,t)$ and electric field $E(x,t)$

Result

- our model:

$$
\begin{aligned}
\partial_t \rho &= -\partial_x(\rho u) \\
\partial_t u &= -u\partial_x u - \frac{1}{\rho}\partial_x P - E \\
\partial_t P &= -u\partial_x P - 3P\partial_x u - 2\partial_x q \\
\partial_t q &= -u\partial_x q - 4q\partial_x u + \frac{3P}{2\rho}\partial_x P - \partial_x\left(\frac{P^2}{2\rho} + \frac{2q^2}{P}\right)
\end{aligned}
$$

- discussions of invariants/conserved quantities