Compression-loose fibre media : continuum constitutive law and single crack case-study

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A particular aspect of fibre network media is that they exhibit a different stiffness in traction and in compression [1]. This nonlinearity is mainly due to the local buckling of fibres. We analyse the global response of a fibre network comprising a pre-existing crack : in this geometry the strain is inhomogeneous and the material nonlinearity dramatically affects the distribution of stress.

We first derive a continuum model representing a 2D network of springs (fibres) whose stiffness in compression is a fraction Î· of the stiffness in extension, with $0 < \eta < 1$. Within the framework of a directional constitutive model, calling $e(\chi)$ the fibre extension in the direction χ and ϕ the local strain anisotropy, we derive a continuous elastic energy density w from integration over all spatial directions as follows [2] :

$$w = \frac{1}{2}k \int_{\omega_t(\phi)} e\left(\chi\right)^2 d\chi + \frac{1}{2}\eta k \int_{\omega_c(\phi)} e\left(\chi\right)^2 d\chi \tag{1}$$

where ω_t (resp. ω_c) stands for the fibre orientation in traction (resp. compression). The elastic energy derived from (1) is of the form $w(\varepsilon, \phi) = \frac{1}{2}\varepsilon^2 f\left(\phi\right)$ where ε is the strain intensity. This equivalent continuum is Hookean for $\eta = 1$ but nonlinear for $\eta < 1$. We are particularly interested in the limit $\eta \to 0$ corresponding to an elastic medium which is soft in compression. The homogenized constitutive law is implemented in a finite element model (FEM) where the derivatives of $w(\varepsilon, \phi)$ are evaluated through an automatic differentiation method (see [3] for details).

We study the influence of the fibre nonlinearity η, and compare simulations of a discrete network of springs to FEM simulations based on the continuous equivalent medium. The stress around the crack tip is analysed and compared with the classical asymptotic from linear elastic fracture mechanics. We also explain the emergence of unloaded area on both sides of the crack.

Then, we highlight the strain-induced anisotropy (orientation of buckled fibres) around the crack [4]. The propagation of this anisotropy when increasing the fibre nonlinearity $\eta \to 0$ is discussed. We find that a diamond shaped area is progressively unloaded (both principal strains are compressive) as $\eta \to 0$.

Références

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