Kinetic description

- VLASOV equation: dynamics of the distribution function $f(\mathbf{x}, \mathbf{v}, t)$ of particles in phase space

$$\frac{\partial f}{\partial t} = -\mathbf{v} \cdot \nabla f - \frac{q}{m} (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \frac{\partial f}{\partial \mathbf{v}} = \{f, \mathcal{H}\}$$

along with MAXWELL's equations for $\mathbf{E}(\mathbf{x}, t)$ and $\mathbf{B}(\mathbf{x}, t)$

- Hamiltonian structure: alternating bracket $\{\cdot, \cdot\}$ which is bilinear and satisfies the JACOBI identity

$$\{F, \{G, H\}\} + \{H, \{F, G\}\} + \{G, \{H, F\}\} = 0$$

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Reductions

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Fluid description

- dynamics in configurations space: $\mathbf{x}$ instead of $(\mathbf{x}, \mathbf{v})$
- infinite number of field variables (density, velocity, pressure, ...)
  $\Rightarrow$ need to close these equations

Objective: obtain closed fluid models that preserve the Hamiltonian structure of the kinetic equations

Framework

- 1D Vlasov-Poisson: $\partial_t f = -v\partial_x f - \Phi' \partial_v f$ and $\Phi'' = \int f \, \mathrm{d}v - 1$
- 3-fields model: density $\rho(x,t)$, velocity $u(x,t)$ and pressure $P(x,t)$

Result

- our model:

$$\partial_t \rho = -\partial_x(\rho u)$$

$$\partial_t u = -u\partial_x u - \frac{1}{\rho}\partial_x P + \Phi'$$

$$\partial_t P = -u\partial_x P - 3P\partial_x u - 2\partial_x q(P/\rho^3)$$

- discussions of invariants/conserved quantities
- application to the drift-kinetic equation

$\Rightarrow$ Leads to the water-bag closure