Faraday wave lattice as an elastic metamaterial

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We use the Faraday instability to shape the fluid-air interface with a regular pattern. This hydrodynamic instability appears at the interface between two fluids subjected to a vertical oscillation [1]. Above a certain threshold of acceleration a_c, the surface shows a stationary deformation both stable in time and regular in space, with a Faraday wavelength λ_F defined by the inviscid gravity-capillary wave dispersion relation $\omega_0^2 = (gk_F + \frac{\sigma}{\rho}k_F^3)\tanh(k_F h)$, where $k_F = (2\pi)/\lambda_F$ is the Faraday wavenumber, g is the acceleration of gravity, σ is the surface tension of the fluid, h the fluid depth and ρ its density.

Upon increasing the driving amplitude to about twice the threshold value, spontaneous oscillations of the square lattice appear [2]. We show that these oscillations are in-plane modulations of the pattern along its two main directions. They occur at a frequency f much lower than the Faraday frequency f_F whereas their spatial wavelength $\lambda \simeq 2.0$ cm is 4 times larger than λ_F. In our experimental conditions and at this frequency f, the gravito-capillary dispersion relation gives a wavelength of $\lambda_{gc} = 23.26$ cm much larger than λ. This means that the transverse standing wave responsible for the pattern oscillations is governed by a different physical mechanism that we identify by performing a new set of experiments.

We investigate all the oscillating modes of the Faraday wave pattern by locally forcing the vibrations of stable square patterns. We add to the vessel a custom-made forcing device consisting of a comb dipping into the liquid to a small depth. The comb oscillates horizontally in the reference frame of the container at frequencies ranging from 0.5 Hz to 5 Hz. We observe a transversal wave that propagates away from the forcing device at the forcing frequency f. From the experimental data we extract its wavenumber k_T for each value of f. The dispersion relation we obtain appears linear, with a phase speed equal to $v_\varphi = 4.60$ cm.s^{-1}. Such a linear dispersion relation reveals the elastic nature of the waves we observe.

We propose a physical mechanism combining surface tension with the Faraday structured interface. We consider the stable Faraday interface as a reference state. When this interface is sheared, its surface increases as well as its energy due to the surface tension σ of the fluid. By comparing this energy difference to the elastic energy of a sheared material, we are able to calculate an effective 2D shear modulus for our interface. The phase velocity we obtain is $c_T = 4.84 \pm 0.63$ cm.s^{-1}, which is in excellent agreement with the experimental result of 4.60 cm.s^{-1}.

This means that we observe the emergence of a new physical property, namely an effective 2D elasticity, at the liquid-air interface. Our interpretation reveals that it is intimately related to the existence of a periodic pattern imprinted on the liquid interface. From this perspective, the Faraday wave pattern creates a mechanical metamaterial [3] at macroscopic scale.

Références

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