Modal stability analysis of mechanically-driven flows in rigid rotating ellipsoids

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Because of gravitational torques generated by their orbital partners, most of planets, moons and stars have ellipsoidal shapes. It also gives birth to mechanical forcings, which lead to time-dependent rotating angular velocities. For instance librations are periodic oscillations about a fixed, mean rotation rate of a body. The fluid response to the librational forcing through topographic coupling in ellipsoids can be important to understand the physics of an orbital body [1]. It has also been proposed that librations can sustain self-sustained dynamos [2], by injecting a part of the huge rotational energy in the system to drive intense fluid flows. Thus librations (and more generally any mechanical forcing) may be a viable alternative to thermo-chemical convection as driving mechanism for planetary dynamos.

We investigate the hydrodynamic global stability of incompressible, inviscid fluids enclosed in rotating rigid ellipsoids. Following [3], three-dimensional perturbations upon a mechanically-driven base state are expanded onto a finite-dimensional polynomial basis, which is an invariant of the governing equation. By combining symbolic and numerical computations, we are able to get the growth rate and the velocity structure of any inertial instability in the linear growth. The method is valid for ellipsoids of arbitrary shape. We extend the work of [2] and perform the stability analysis of the experiment of [4], considering spheroidal containers undergoing longitudinal librations. We predict new instabilities of high spatial complexity, some of them which may be observed in laboratory experiments. New instabilities are also found for other mechanical forcings.

Finally, we outline how to use the inertial modes to study the weakly non-linear saturation of any inertial instability. Indeed, the underlying mechanism of inertial instabilities is a triadic resonance between the mechanically-driven base flow and a pair of inertial modes of the system. With this new approach, we aim at giving a quantitative explanation of the saturation with a low-dimensional weakly nonlinear model of inertial instabilities, valid for both laboratory and numerical experiments. The method will be applied first to a given librational configuration [4;5].

Bibliography

[1] Vantieghem, S., Cébron, D., and Noir, J. (2015). *Journal of Fluid Mechanics*, 771:193–228.
[2] Wu, C. C. and Roberts, P. H. (2013). *Geophysical & Astrophysical Fluid Dynamics*, 107:20–44.
[3] Wu, C. C. and Roberts, P. H. (2011). *Geophysical & Astrophysical Fluid Dynamics*, 105:287–303.
[4] Grannan, A., Le Bars, M., Cébron, D., and Aurnou, J. (2014). *Physics of Fluids (1994-present)*, 26(12):126601.
[5] Favier, B., Grannan, A., Le Bars, M., and Aurnou, J. (2015). *Physics of Fluids (1994-present)*, 27(6):066601.