Observation of Random Riemann Waves In Integrable Turbulence

F. Gustave , P. Suret, G.A. El, S. Randoux

*"The field of **integrable turbulence** deals with the general question of **statistical changes** that are experienced by **ensembles of nonlinear random waves** propagating in systems ruled by **integrable equations**."*

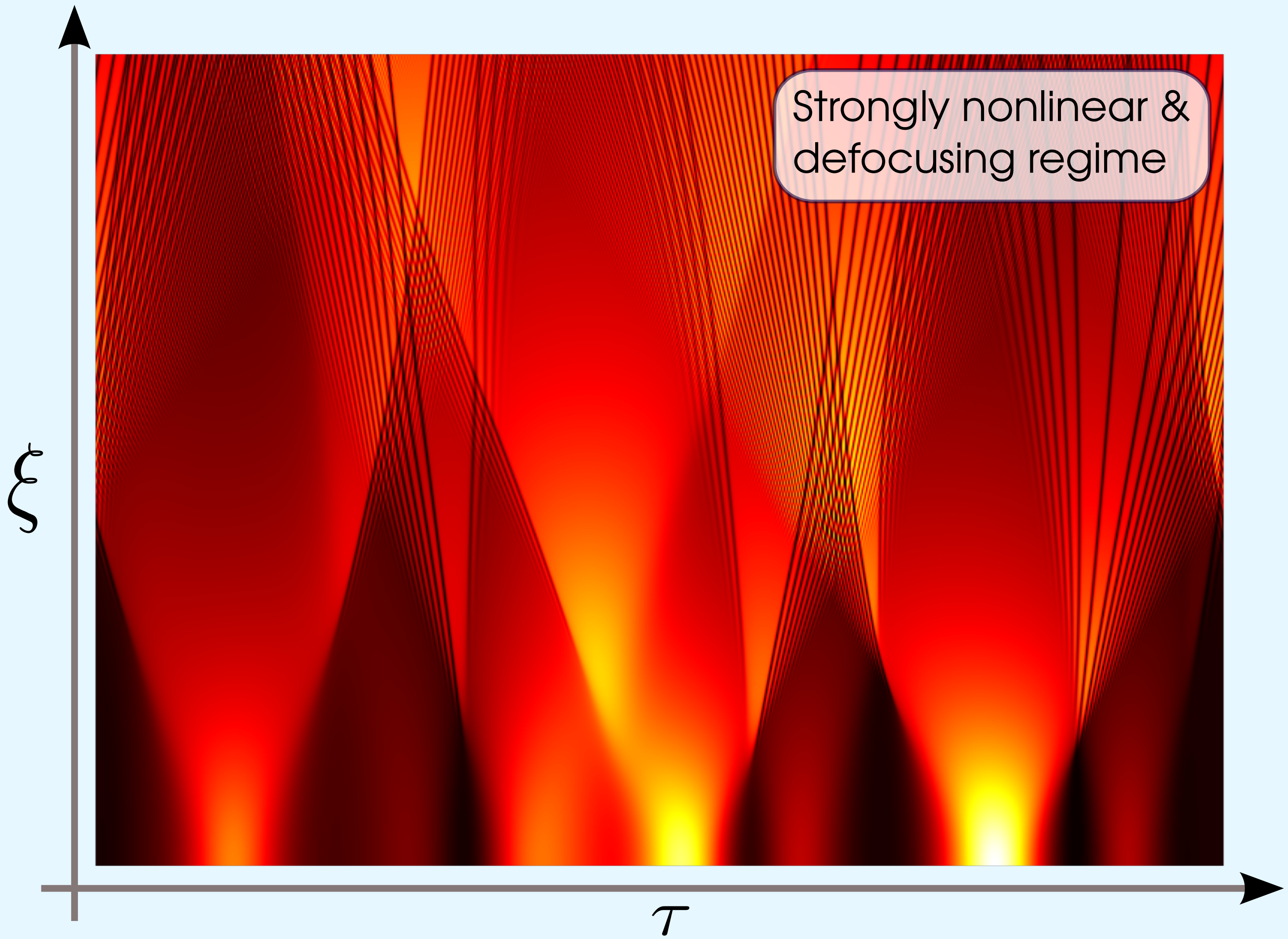
V.E. Zakharov, Stud. Appl. Math. 122, 219-234 (2009)

In optical fibers ?

$$i\epsilon\frac{\partial\psi}{\partial\xi}+\frac{\epsilon^2}{2}\frac{\partial^2\psi}{\partial\tau}-|\psi|^2\psi=0$$

- $\psi(\xi,\tau)$: Optical field (complex)
- 1D Nonlinear Schrodinger equation (1D-NLSE) in the semiclassical limit ($\epsilon \to 0$)

$$\epsilon=\sqrt{L_{NL}/L_D}$$



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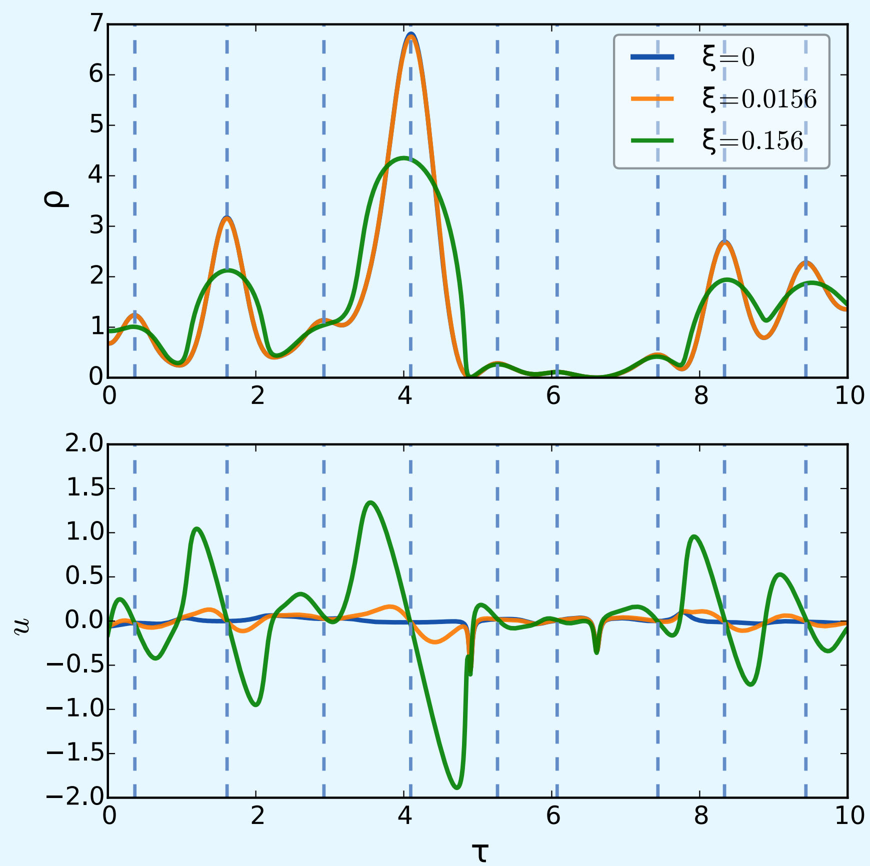
Université de Lille 1 Sciences et Technologies

CEMPI Centre Européen pour les Mathématiques, la Physique et leurs Interactions

PhLAM



Madelung transformation : $\psi(\xi,\tau) = \sqrt{\rho(\xi,\tau)}e^{i\frac{\phi(\xi,\tau)}{\epsilon}}$

$$\rho_\xi + (\rho u)_\tau = 0$$
$$u_\xi + u u_\tau + \rho_\tau = 0$$

$$\rho(\xi,\tau) = |\psi(\xi,\tau)|^2$$
$$u(\xi,\tau) = \frac{\partial \phi}{\partial \tau}$$

Shallow water waves

Rieman invariants : $r_{1,2}(\xi,\tau) = u \pm 2\sqrt{\rho}$

Riemann waves equation

$$\frac{\partial r_i}{\partial \xi} + \frac{r_i}{2}\frac{\partial r_i}{\partial \tau} = 0, \quad i = 1,2.$$