Steady streaming in standing waves

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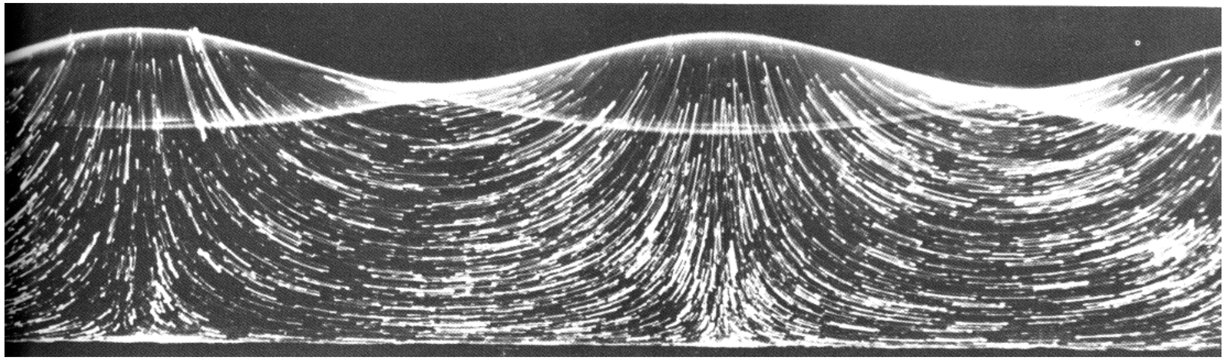
- **standing waves : trajectories**



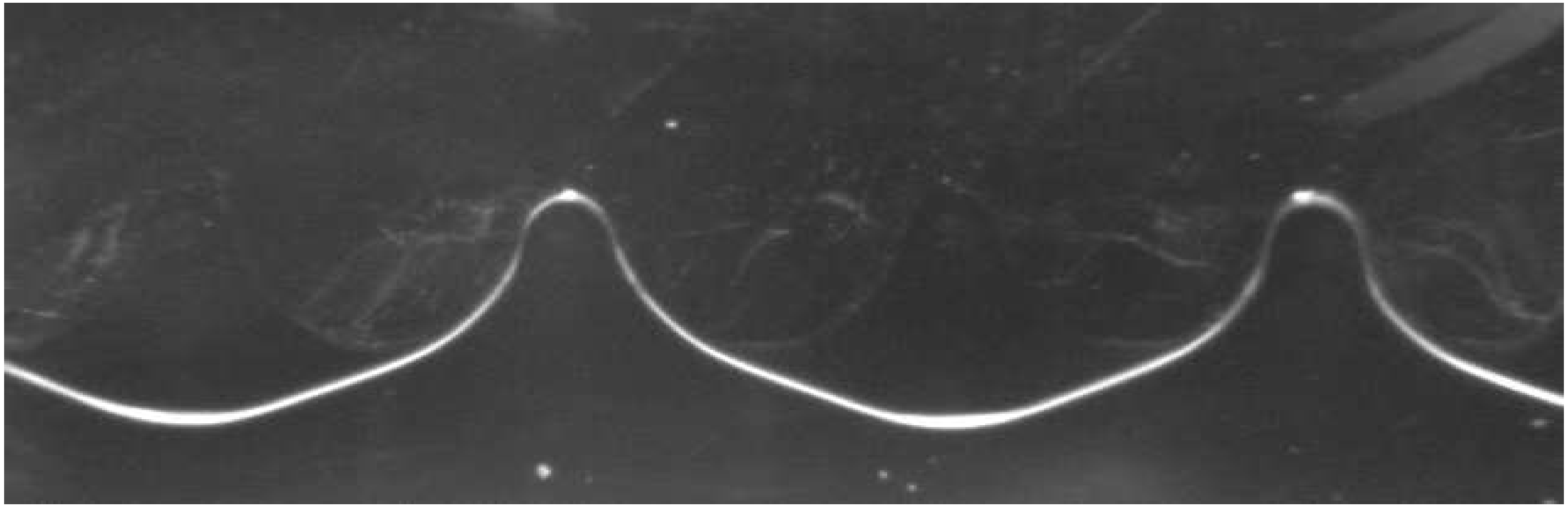
Van Dyke, *an album of fluid motion*,1982.

Velocities (linearized potential solution)

$$u = A\omega\,[\cosh k(h - z)\,/\,\sinh kh]\,\cos\omega t\,\cos kx$$

$$v = A\omega\,[\sinh k(h - z)\,/\,\sinh kh]\,\cos\omega t\,\sin kx$$

Strobed picture, synchronous of the wave period



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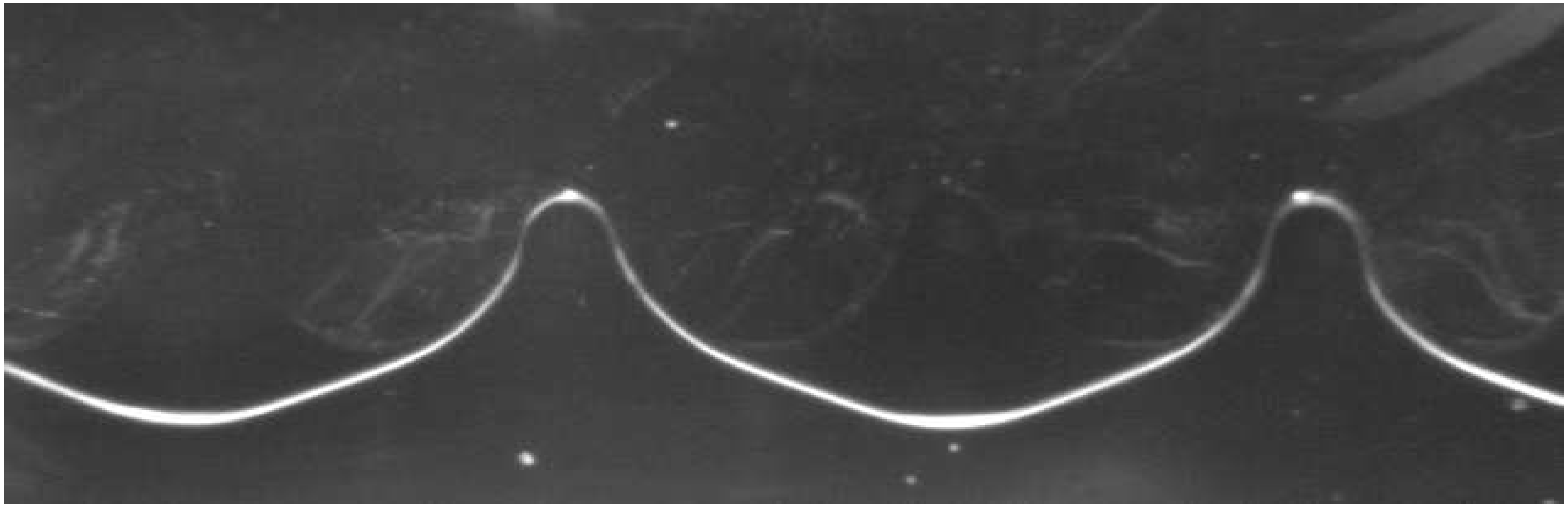

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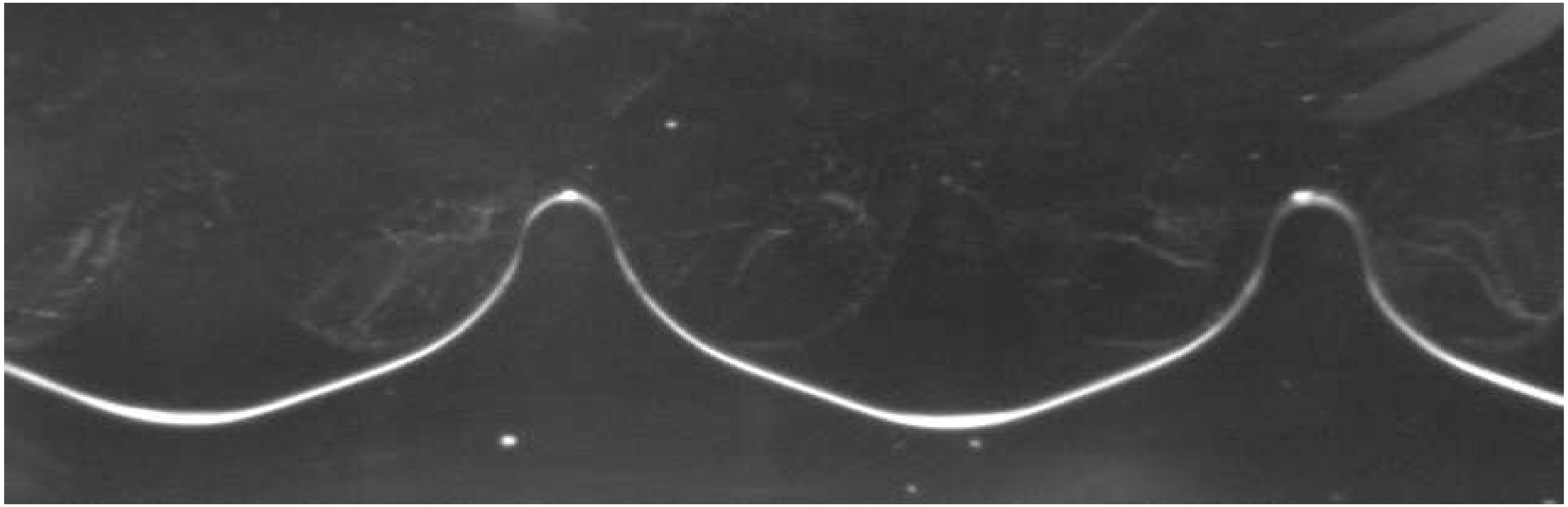

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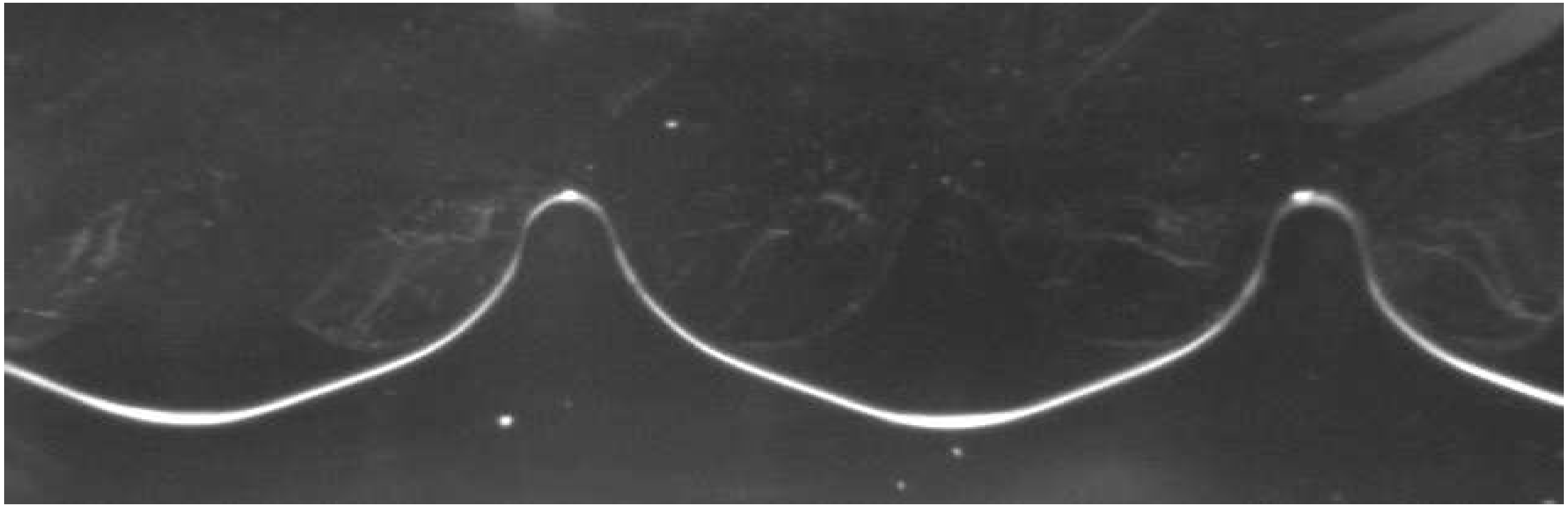

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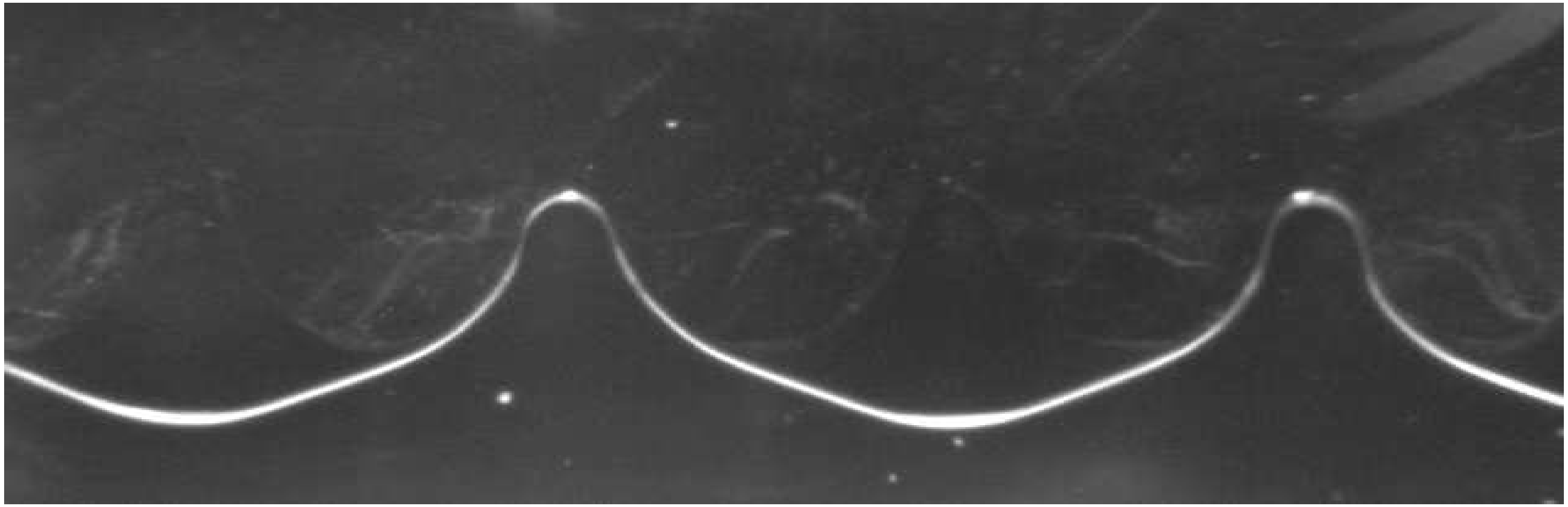

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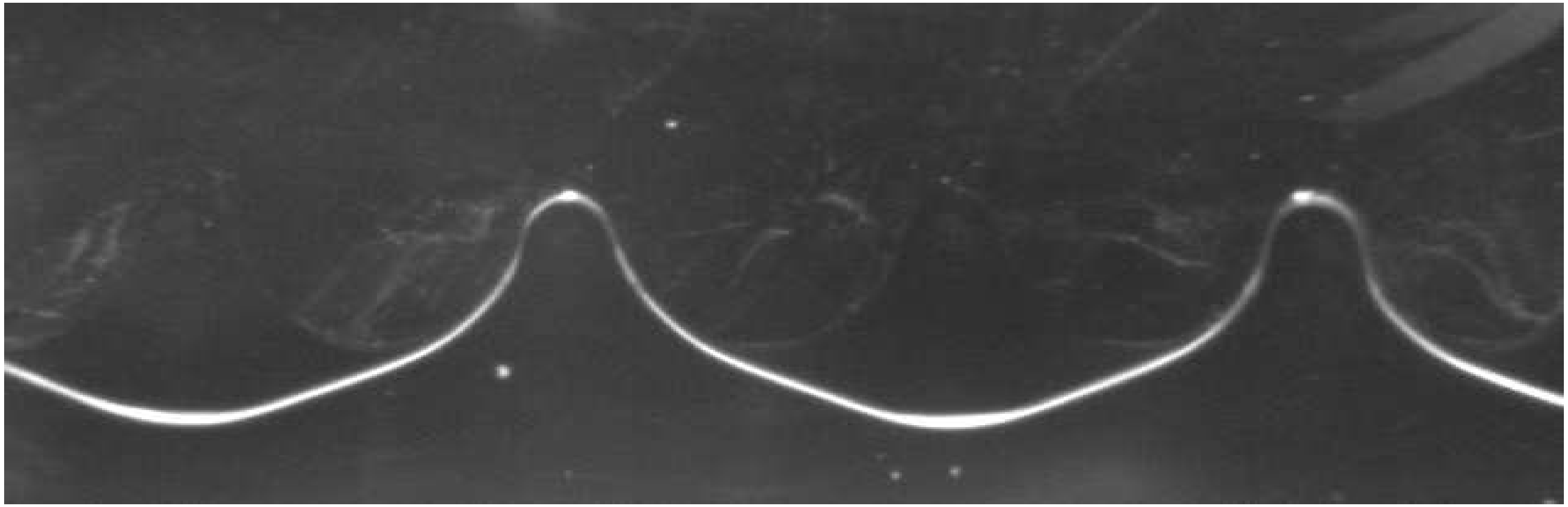

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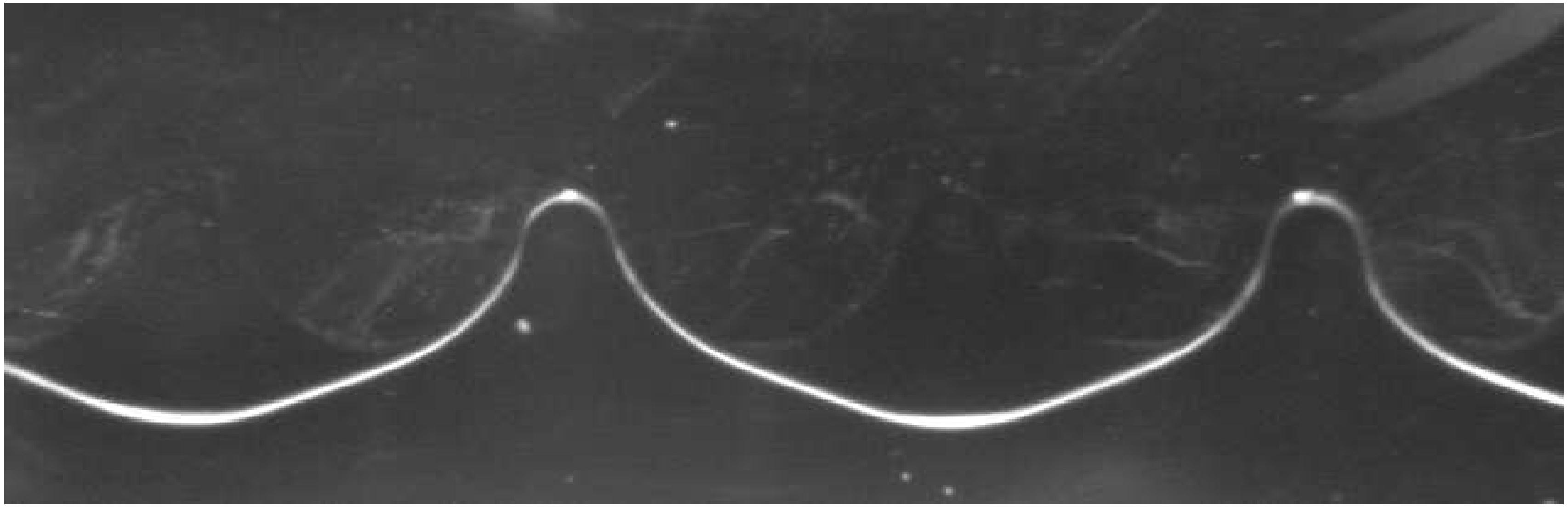

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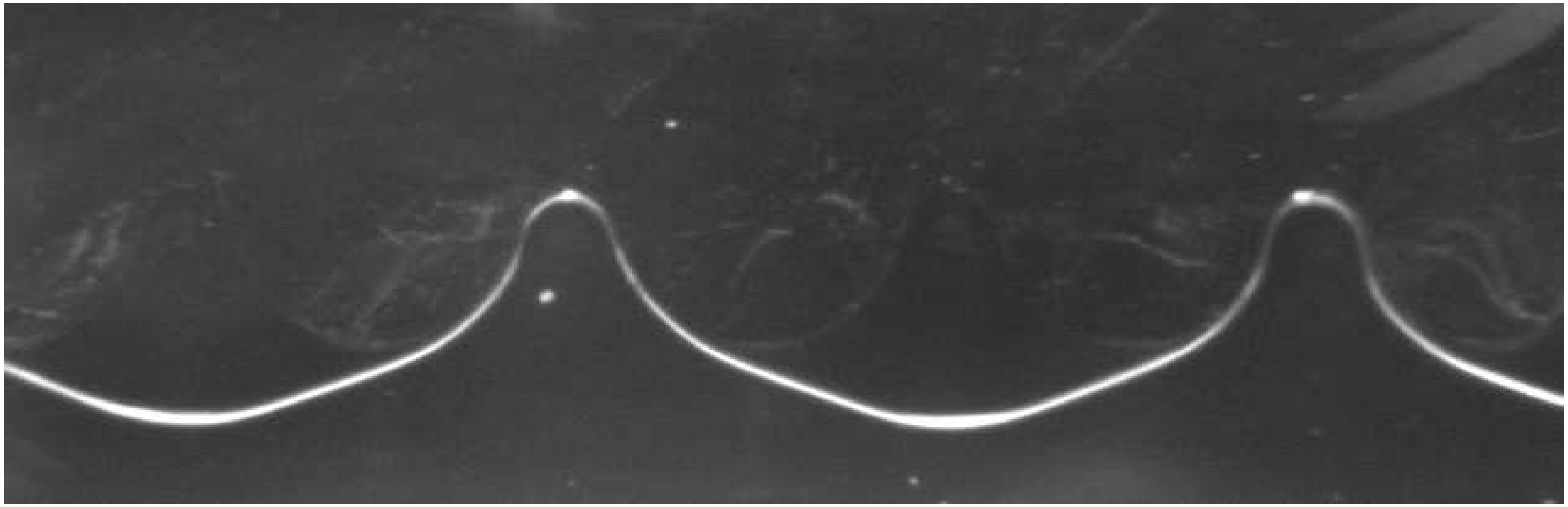

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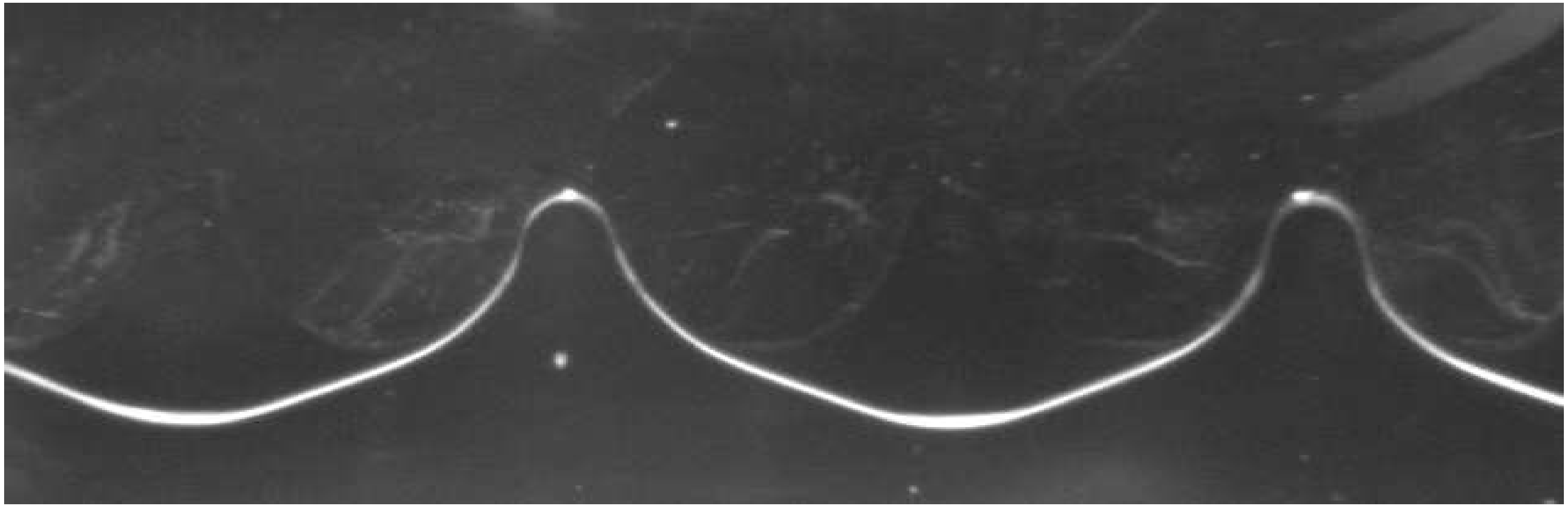

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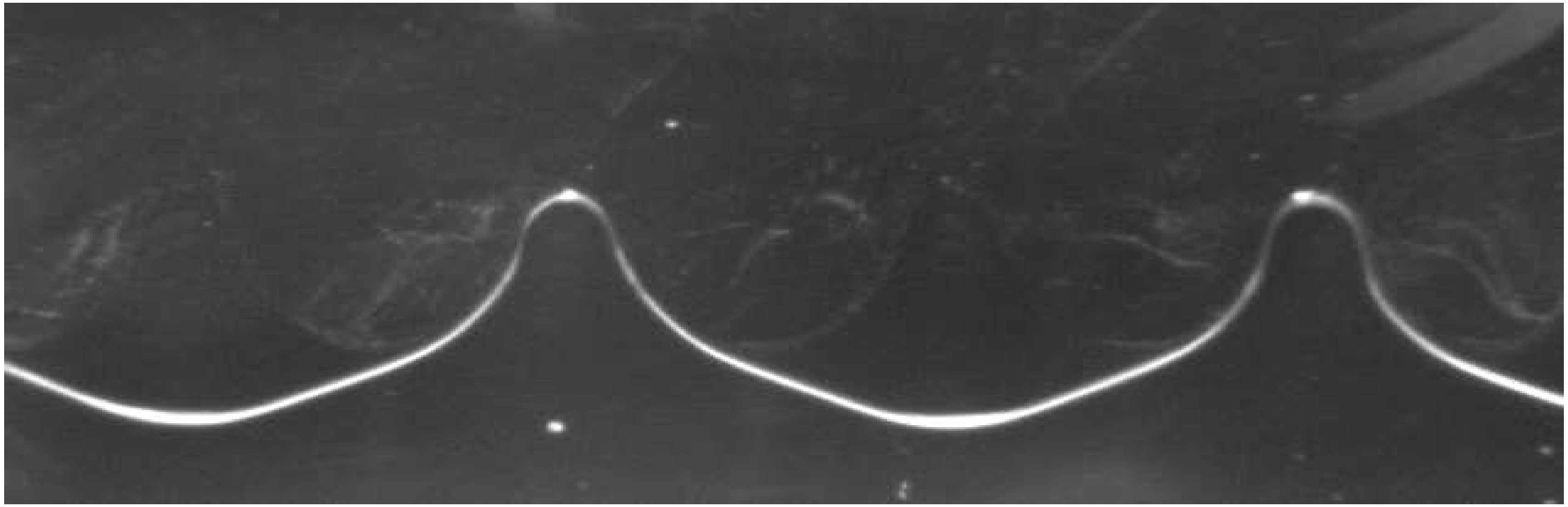

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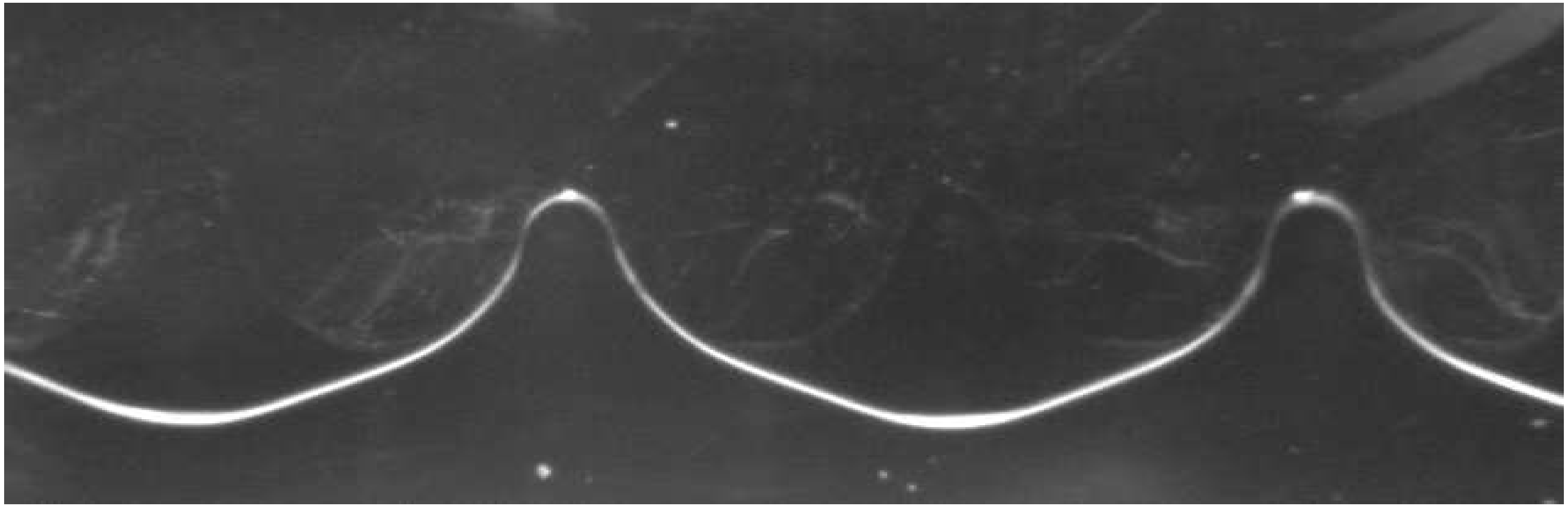
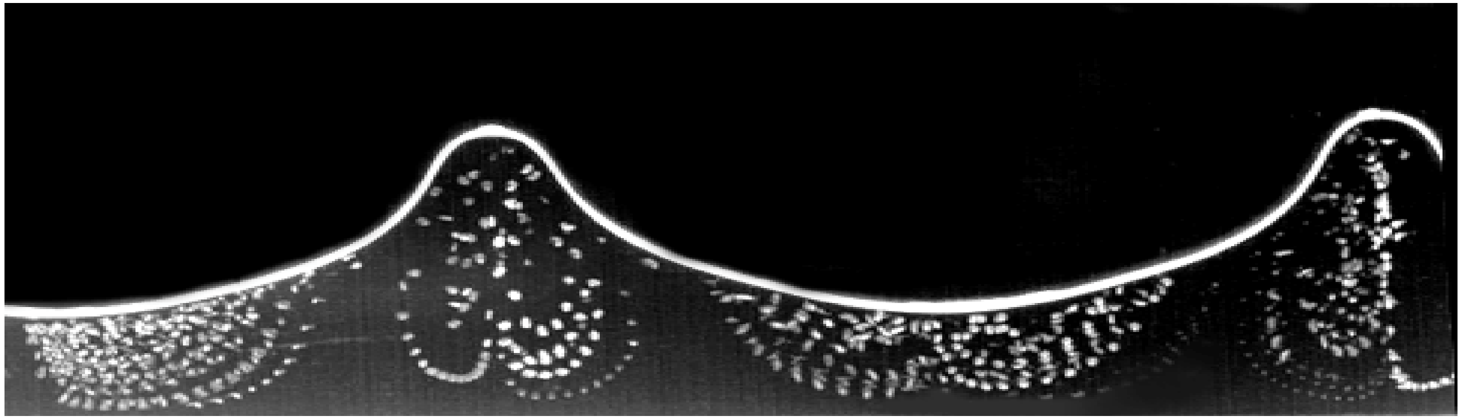


Recirculation eddies



• Period of the (Eulerian) velocity field = wave period
Period of the velocity attached to one particle ~ 10^2 wave periods !

• two couples of contrarotating eddies by wavelength
i.e. wavelength of the recirculating cells = λ / 2

Minimal ingredients for the modelisation of the recirculation eddies

- **take viscosity into account**

- **take nonlinearities into account**

$\longrightarrow$ time-averaged motion $\neq$ zero

- **Trajectories : integration of the Lagrangian velocities**

Nonlinear effects and stready components

At the free-surface: the zero shear-stress condition is a source of vorticity.

$$\underbrace{\frac{\partial \mathbf{\Omega}}{\partial t}}_{\omega\Omega} + \underbrace{(\boldsymbol{v} \cdot \nabla)}_{A^2k^2\omega^2} \mathbf{\Omega} = \underbrace{\nu \nabla^2}_{\text{Diffusion}} \mathbf{\Omega}$$

Helmoltz Eq. (in 2d)

- The nonlinear term advects this vorticity far beneath the boundary layer. This gives rise to a steady vorticity of second order in the bulk

$$\Omega \propto A^2k^2\omega$$

- The wavelength of the nonlinear term $(\boldsymbol{v} \cdot \nabla)\mathbf{\Omega}$ is λ / 2

To get the (2nd order) stream function associated to the mass transport, we have to consider the (2nd order) Lagrangian velocity

$$\mathbf{U}_2 = \mathbf{u}_2 + \int_0^t \mathbf{u}_1 \, \mathrm{d}t \cdot \mathrm{grad}\, \mathbf{u}_1$$

and to average it over one period

$$\Psi = \overline{\psi_2} + \overline{\int \frac{\partial \psi_1}{\partial y} \mathrm{d}t \, \frac{\partial \psi_1}{\partial x}}$$

We obtain thus

$$U \approx -\frac{A^2 k\omega}{2} \sin(2kx) \exp(2ky)$$

$$V \approx -\frac{A^2 k\omega}{2} \cos(2kx) \exp(2ky)$$

Conclusion

- **Experimental visualization** of **steady secondary flows** in **standing waves**.

- Up to now, the boundary condition **no shear stress at the free surface** (i.e. *no steady shear)* had been considered as **incompatible** with development of **steady streaming eddies** in the case of infinite depth

- **On the contrary,** we have shown both experimentally and theoretically that the surface **boundary conditions are the cause** of the formation of the recirculation rolls.