Many phenomena in nature involve motion of viscous flows, which are widely believed to be described by Navier-Stokes equations (NSE). These equations are used for instance in numerical simulations of flows in astrophysics, climate or aeronautics. These equations are the cornerstones of many physical and engineering sciences, and are routinely used in numerical simulations. From a mathematical point of view, however, it is still unclear whether the Navier-Stokes equations are a well-posed problem in three dimensions, i.e. whether their solutions remain regular over sufficient large time or develop singularities. Historically, the search for singularities in NSE was initiated by Leray who introduced the notion of weak solutions (i.e. in the sense of distribution). This notion was used to prove that the mathematical singular set has a one-dimensional Hausdorff measure equals to zero in space-time. Therefore, if these singularities exist, they must be extremely localized events in space and time. This makes their direct detection an outstanding problem. For some times, the best suggestive evidence of their existence was provided by the observation that the energy dissipation rate in turbulent flows tends to a constant at large Reynolds numbers. This observation is at the core of the 1941 Kolmogorov theory of turbulence, and was interpreted by Onsager as the signature of singularities with local scaling exponent $h = 1/3$. Later, it was conjectured that the singularities are organized into a multifractal set. Analysis of measurements of 3D numerical or 1D experimental velocity fields showed that the data are compatible with the multifractal picture, with a most probable $h$ close to $1/3$. However, this analysis could not reveal any information on the space-time statistics of (possible) singularities.

A major breakthrough was achieved when Duchon and Robert performed a detailed energy balance for weak solutions of INSE, and compute the contribution stemming from an eventual lack of smoothness. They show that it can be lumped into a single term, that quantifies the "inertial" energy dissipation, i.e. the energy dissipated by non-viscous means. The purpose of this talk is to discuss how this mathematical result can be used for high spatial resolution measurements of the velocity field in an experimental turbulent swirling flow to infer properties of the energy dissipation in a turbulent flow. First, we show that our results are consistent with the existence of quasi-singularities even at the dissipative scales of turbulence. We show that they are very intermittent in space and time and provide the first experimental attempt at characterization of extreme events of inertial-dissipation.

Références