Turbulence forte et faible dans le model MMT

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Many physical phenomena are associated with the propagation of dispersive waves. While in some cases their dynamics is linear, many relevant situations manifest a non-negligible nonlinearity which produces complex patterns. When the number of degrees of freedom is large enough, such problems must be treated in a statistical manner. The Weak-Wave Turbulence (WWT) theory is a very general framework by which the statistical properties of a large number of incoherent and interacting waves can be studied [1,2,3,4].

The WWT theory has been applied to a variety of fields such as for example ocean waves, capillary waves, Alfvén waves or optical waves. Despite the beauty of these theoretical results it is of paramount importance to verify if the assumptions behind the theory are realized in practice and thus if this approach is suitable to address physical issues of complex wave systems.

Almost 20 years ago a family of one-dimensional nonlinear dispersive wave equations, namely the MMT model [5], was introduced as a, in principle simple, model for assessing the validity of WWT theory ; however, the results reported in [5] were somehow discouraging and it was reported that "the predictions of weak turbulence theory fail and yield a much flatter spectrum compared with the steeper spectrum observed in the numerical statistical steady state". The MMT model has become a "paradigm-like" for the verification of the WWT predictions, even though it is a one-dimensional idealised model. Since then many examples of deviations from WWT scalings have been found, a signature of "intermittency". Notably in different experiments of mechanically forced surface gravity waves by different groups. Nonetheless the origin of such intermittency remains mysterious.

In this work, we consider the MMT model in the defocusing regime with gravity-wave dispersion as a basic tool for studying the properties of wave turbulence. In particular, we show that in the direct energy cascade regime the WWT theory offers an accurate prediction for wave spectral slope, provided the ratio between the nonlinear to the linear Hamiltonian is sufficiently small. When the ratio of nonlinear to linear energy is large enough, the prediction of the wave turbulence fails and, as shown in [5], the spectrum becomes steeper than the WWT prediction. The calculation of the structure functions and of the probability density function (PDF) of increments of the wave field at different scales show that, while in the strongly nonlinear regime the dynamics is characterised by intermittency, in the weakly nonlinear case described by the WWT spectrum the wave field exhibits a quasi-self-similar scaling. The model cannot describe the phenomenon of wave breaking or the formations of cusps ; therefore, it offers a unique tool to establish that the observed intermittency cannot be attributed solely to such singular or quasi-singular structures.

Références

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