Violation of the gyroptropic pressure closure due to a velocity shear in a magnetised plasma

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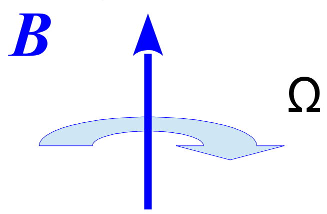
- The presence of a magnetic field induces a gyrotropic form of the pressure tensor for ω << Ω



$$\Pi_{ij} = \begin{pmatrix} P_\perp & 0 & 0 \\ 0 & P_\perp & 0 \\ 0 & 0 & P_{||} \end{pmatrix}$$

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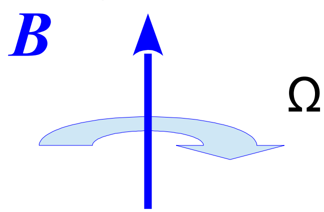

$$\Pi_{ij} = \begin{pmatrix} P_\perp & * & * \\ * & P_\perp & * \\ * & * & P_\parallel \end{pmatrix}$$

- However he *occurrence of non-groytropic particle distributions that are anisotropic in temperature/pressure is observed directly by satellites* in the solar wind and magnetosphere, and indirectly by the signature of anisotropy-driven instabilities in space plasmas.

- We look for a *dynamical anisotropisation mechanism* by inspecting the *full pressure tensor equation*, obtained by taking the second order velocity moment of Vlasov equation.

$$\frac{\partial \mathbf{\Pi}^\alpha}{\partial t} + \nabla \cdot (\boldsymbol{u}^\alpha \mathbf{\Pi}^\alpha) + (\nabla \boldsymbol{u}^\alpha) \cdot \mathbf{\Pi}^\alpha + ((\nabla \boldsymbol{u}^\alpha) \cdot \mathbf{\Pi}^\alpha)^T + \nabla \cdot \boldsymbol{Q}^\alpha = \Omega_\alpha (\mathbf{\Pi}^\alpha \times \boldsymbol{b} + \boldsymbol{b} \times \mathbf{\Pi}^\alpha)$$

Approach and results

- The gradient tensor apppearing in the full pressure tensor equation (second order moment of Vlasov equation) *can induce anisotropic deformation on the pressure tensor*

- In a 2D geometry with $\boldsymbol{\omega}$ x $\boldsymbol{B} = 0$ the *gyrotropic* and *non-gyrotropic anisotropies* evolve as:

$$A^{ng} \equiv \frac{P_1 - P_2}{P_1 + P_2} = \frac{\sqrt{(\mathrm{tr}[\mathbf{\Pi}_\perp])^2 - 4\det[\mathbf{\Pi}_\perp]}}{\mathrm{tr}[\mathbf{\Pi}_\perp]}$$

$$\frac{dA^{ng}}{dt} = 2D_\perp\left[(A^{ng})^2 - 1\right]\cos[2(\theta - \phi)]$$

$$A^{gyr} \equiv \frac{2P_3}{P_1 + P_2} = \frac{2P_{||}}{\mathrm{tr}[\mathbf{\Pi}_\perp]}$$

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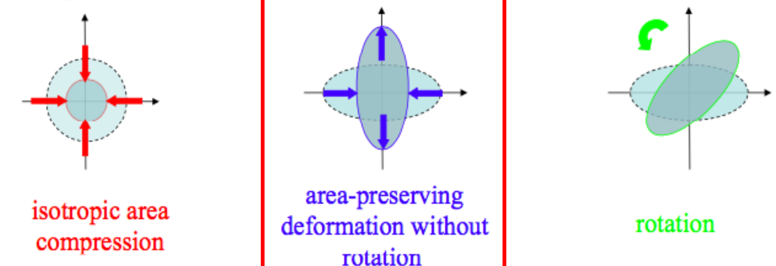
$$\mathbf{\Pi} = \mathbf{\Pi}_\perp + P_{||}\boldsymbol{bb}$$

$$\underbrace{(\nabla\mathbf{u})_\perp}_{\partial_i u_j,\ i=x,y} = \underbrace{-\frac{1}{2}(\nabla\cdot\mathbf{u}_\perp)\mathbf{I}_\perp}_{\mathcal{C}_\perp\delta_{ij}} + \underbrace{\left[\frac{1}{2}\left((\nabla\mathbf{u})_\perp + (\nabla\mathbf{u})_\perp^T\right) + \frac{1}{2}(\nabla\cdot\mathbf{u}_\perp)\mathbf{I}_\perp\right]}_{\mathbf{D}_{\perp,ij}} + \underbrace{\frac{1}{2}\left((\nabla\mathbf{u})_\perp - (\nabla\mathbf{u})_\perp^T\right)}_{\mathbf{W}_{\perp,ij}}$$

	$\mathcal{C}_\perp\delta_{ij}$	$\mathbf{D}_{\perp,ij}$	$\mathbf{W}_{\perp,ij}$
	Rate of Isotropic Compression	*Trace-less Rate of Shear*	*Vorticity*



Action on $\mathbf{\Pi}_\perp$	$\mathcal{C}_\perp\mathbf{\Pi}_\perp$	$\{\mathbf{D}_\perp, \mathbf{\Pi}_\perp\}$	$[\mathbf{W}_\perp, \mathbf{\Pi}_\perp]$

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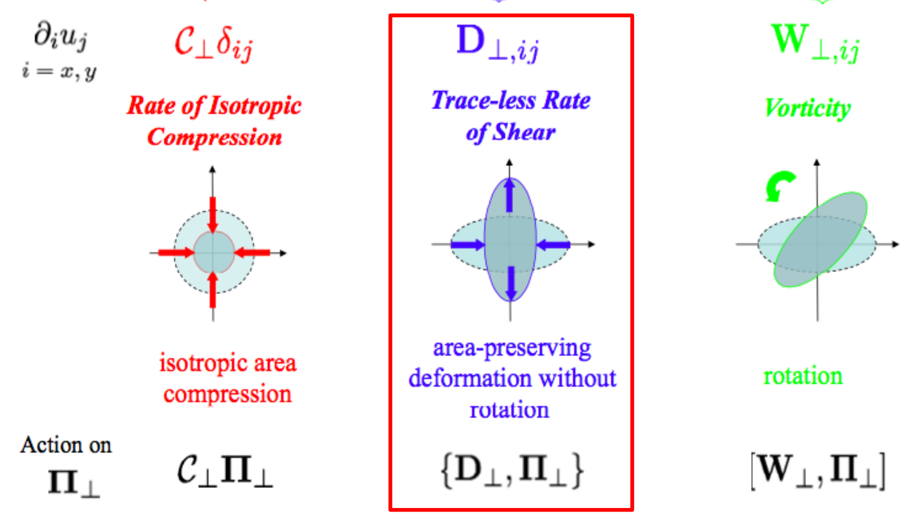
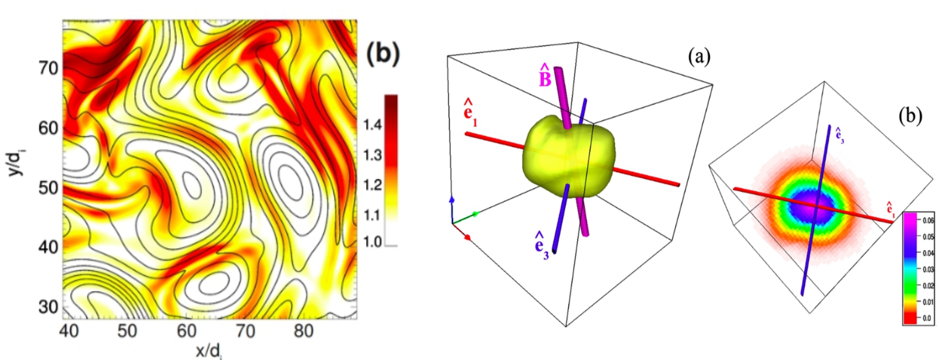
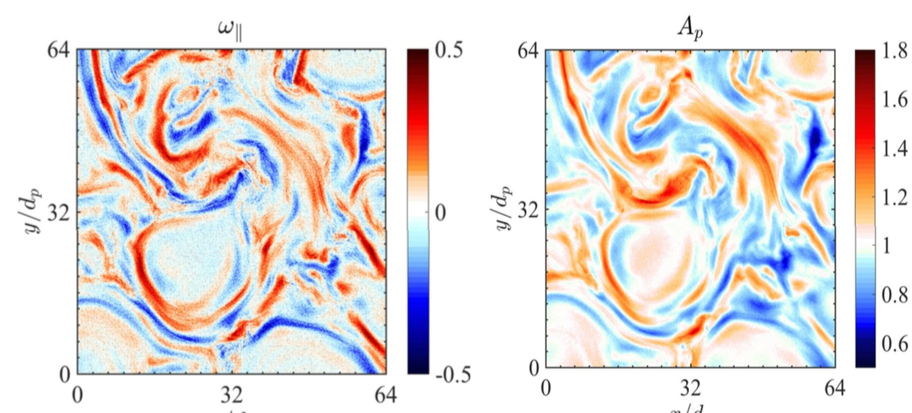
$$(\nabla\mathbf{u})_\perp = -\frac{1}{2}(\nabla\cdot\mathbf{u}_\perp)\mathbf{I}_\perp + \left[\frac{1}{2}\left((\nabla\mathbf{u})_\perp + (\nabla\mathbf{u})_\perp^T\right) + \frac{1}{2}(\nabla\cdot\mathbf{u}_\perp)\mathbf{I}_\perp\right] + \frac{1}{2}\left((\nabla\mathbf{u})_\perp - (\nabla\mathbf{u})_\perp^T\right)$$



- This *shear-induced anisotropisation* mechanism is *relevant to any low-collision/non-viscous fluids* and can help understand *results of Vlasov simulations of Alfvenic turbulence*



Figures from [Servidio et al., PRL 2012]



Figures from [Franci et al., AIP Conf. Proc. 2016]