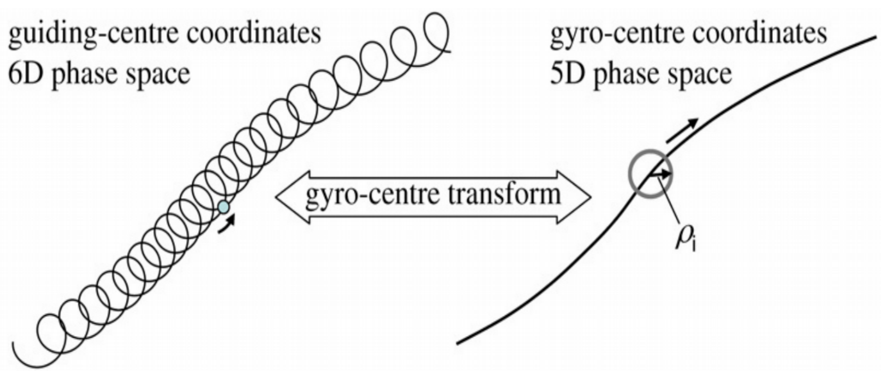
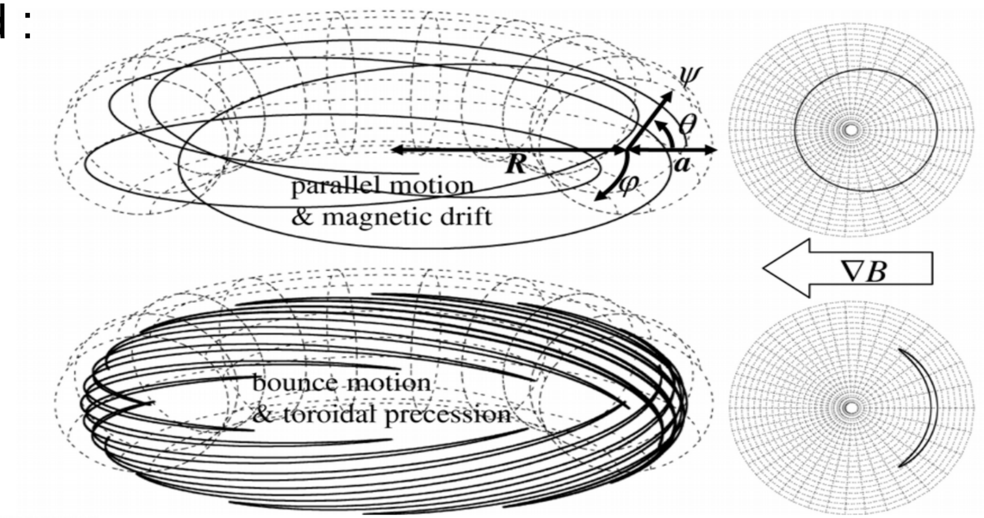
Reduced model of turbulence for magnetized plasmas : shell model and logarithmic discretization

Shaokang XU, Pierre Morel, Özgür Gürcan
Laboratoire de Physique des Plasmas(LPP), École Polytechnique, France

Trapped particles motion, strong magnetic field :





Vlasov equation + averages : $f\,(\alpha,\ \mu,\ E,\ \psi)$

$$\frac{\partial f_{s,\mathbf{k}}}{\partial t} - i\mathbf{k}_{\alpha}\mathcal{J}_{0s}\phi_{\mathbf{k}}\frac{\partial F_s}{\partial \psi} + \frac{E\Omega_d}{Z_s} i\mathbf{k}_{\alpha} f_{s,\mathbf{k}} - \sum_{\mathbf{p}+\mathbf{q}=\mathbf{k}} \left(\mathbf{p}_{\psi}\mathbf{q}_{\alpha} - \mathbf{p}_{\alpha}\mathbf{q}_{\psi}\right)\mathcal{J}_{0s}\phi_{\mathbf{p}} f_{s,\mathbf{q}} = 0$$

Quasi-neutrality equation :

$$c_{\mathbf{k}}\phi_{\mathbf{k}} = \int \left(\mathcal{J}_{0i} f_{i,\mathbf{k}} - \mathcal{J}_{0e} f_{e,\mathbf{k}}\right)\sqrt{E}\,dE$$

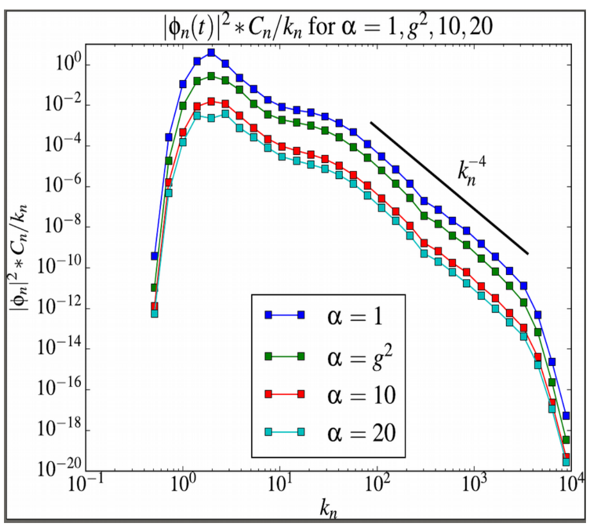
Shell model : truncation to neighbouring modes $\{(n-2,n-1)\,;(n-1,n+1)\,;(n+1,n+2)\}$

$$\frac{\partial f_n^s}{\partial t} = i k_n \mathcal{J}_{0s}^{n} \phi_n \frac{\partial F_s(\psi)}{\partial \psi} - i\frac{E\Omega_d}{Z_s} k_n f_n^s + D$$
$$+\alpha k_n^2 g^{-3}\left(\mathcal{J}_{0s}^{n-2}\phi_{n-2} f_{n-1}^s - \mathcal{J}_{0s}^{n-1}\phi_{n-1} f_{n-2}^s\right)$$
$$-\alpha k_n^2 g^{-1}\left(\mathcal{J}_{0s}^{n-1}\phi_{n-1}^* f_{n+1}^s - \mathcal{J}_{0s}^{n+1}\phi_{n+1} f_{n-1}^{s*}\right)$$
$$+\alpha k_n^2 g\left(\mathcal{J}_{0s}^{n+1}\phi_{n+1}^* f_{n+2}^s - \mathcal{J}_{0s}^{n+2}\phi_{n+2} f_{n+1}^{s*}\right)$$

Isotropy assumption

Different phase approximations :
- Sabra vs GOY

Power law



Kinetic spectrum