Wave propagation through contact-based elastically asymmetric materials

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In the first part, we introduce a class of architectured materials with internal contacts. Among other interesting physical and mechanical properties, internal contacts can ensure tunable contrast in elastic constants in tension and compression. For example, rocks and concrete are elastically asymmetric materials because of multiple internal cracks. In compression, the cracks are closed and fully bear the load, whereas in tension, they are open resulting in a smaller effective bulk modulus compared to compression. However, elastic contrasts found in nature are relatively small. We construct a series of elemental cells, which demonstrate strong elastic asymmetry in tension/compression or in left/right shear. The former materials can be either stiff-in-compression and soft-in-tension, or, vice versa, soft-in-compression and stiff-in-tension. The contrast can be accurately tuned combining different internal cuts and materials of different stiffness [1].

In the second part, we study propagation of elastic waves through a rod with elastically asymmetric behavior emerging from designed elemental cells with internal contacts. Viscoelastic Kelvin-Voigt rheology is used [2]. The resulting governing equation takes the following form [3] :

$$\rho u_{,tt} = E(u_{,x} + \alpha|u_{,x}|)_{,x} + \mu u_{,xxt}, \quad 0 < \alpha < 1, \tag{1}$$

where ρ is the mass density (kg/m^3), E is the elastic modulus (Pa), and μ is the viscosity (Pa·s). The adimensional factor α determines the material asymmetry : the elastic modulus is equal to $E^+ = E(1+\alpha)$ and $E^- = E(1-\alpha)$ in tension and compression, respectively. Thus, the tensile and compressive components of elastic waves propagate at celerities $c^\pm = \sqrt{E^\pm/\rho}$, respectively. Because of this difference, if a harmonic oscillation is induced in such medium, faster tensile wave components catch up with slower tensile wave components and overlap. This overlap leads to emergence of high-frequency oscillations, which are efficiently damped. This damping is enhanced by local sub-harmonic resonances, which are present in materials with discontinuous elastic characteristics [4].

Some arrangement of asymmetric sections enable to filter out compressive or tensile signal components, other arrangements permit to damp efficiently the whole signal. Depending on the contrast parameter α, the damping layer can be chosen relatively small compared to the wavelength of the incident wave.

Références

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