Propagation of dipoles in non-linear elastic media

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How do forces propagate through complex media such as the cytoskeleton?

Previous numerical works proved that force dipoles applied to non-linear networks of fibers are not simply propagated. At large distances from the point of application of the forces, the dipole is amplified and rectified towards an effective contractile dipole independent on whether the local dipole is contractile or extensile [1]. This effect is due to the non-linear response of fibers softening under high compression thus giving rise to an effective rope-like medium.

We generalize the finding to all isotropic materials distorted by a local dipole. It turns out that also neutral dipoles (i.e. neither contractile nor extensile) are rectified. Intuitively, the rectification direction depends on the medium behavior: a strain-stiffening medium rectifies to an effective contractile dipole, but a strain-softening one rectifies to an extensile dipole.

For cells, an important consequence of this finding is that the cytoskeleton may actively regulate and "organize" large scales forces from disorganized local forces, without invoking a small-scale self-organizing principle. The possibility to tune the material response means that both contractility and extensility may be controlled by a single network property (i.e. the connectivity).

However, two fundamentally different non-linear contributions are systematically present in all non-linear media: geometrical non-linearities and material non-linearities. The geometrical terms are tuned by the Poisson ratio and not by the material non-linerities. Material non-linearities instead, describing e.g. the strain-stiffening or -softening, can be tuned to obtained the desired effective dipole. The consequence is a counterintuitive shift of the threshold from contractile to extensile materials.

Our results are validated by finite elements integration of a generic hyper-elastic model interpolating between strain -stiffening and -softening response. This shows that the results are robust, and may be inspire future material design or applied in different contexts.

References

[1] Ronceray, P., Broedersz, C. P. and Lenz, M. Fiber networks amplify active stress. *Proc. Natl. Acad. Sci.* **113**, 2827–2832 (2016).