Critical transitions in geometrically constrained incompressible turbulence

Adrian van Kan[1] & Alexandros Alexakis[2]

Laboratoire de Physique Statistique, École Normale Supérieure, Université Pierre et Marie Curie, 24 rue Lhomond, 75005 Paris
adrien.van.kan@ens.fr

Abstract. Geophysical and astrophysical flows are often subject to geometrical constraints such as thinness in a particular direction. Geometrical constraints strongly affect the nature of flow at high Reynolds numbers Re. This is related to the well-known fact that the behaviour of flows at large Re is depends on the dimensionality of the system. In the two-dimensional Navier-Stokes equations, conservation of enstrophy in addition to energy gives rise to an inverse energy cascade, a transfer of energy to the large scales, while in three dimensions, vortex stretching transfers energy to small scales in a direct cascade. For the idealised case of forced incompressible three dimensional flow in a triply-periodic box with dimensions $L \times L \times H$, with spectrally local forcing at k_f at fixed energy injection rate, it has been found that for high Re and small $A = H/L$, a transition occurs when $S = k_f H$ is decreased below $S_c \approx 0.5$, [1,2,3]. For $S > S_c$, there is three-dimensional turbulence with a purely forward cascade, while for $S < S_c$, an inverse cascade spontaneously emerges. Similar transitions have been found as a function of Rossby number Ro when rotation is added, [4,5]. The inverse cascade leads to a growth of total energy at large scales. Even in the absence of large scale dissipation mechanism this process saturates at late times leading to the formation of a condensate. In two-dimensional turbulence, the turbulent condensate is well understood, [6], but in the case of thin three-dimensional layers the behaviour of the condensate phase has not yet been investigated.

In this work we study turbulence in thin layers in the condensate state using a large number of direct numerical simulations varying all parameters of the system. We investigate the energy budget in large and small scales as a function of Re, S and the aspect ratio A. It is shown that in a range of $S < S_c$, an effective eddy viscosity-type spectrally non-local transfer of energy is responsible for the saturation of the condensate. For even smaller S, the flow is entirely two-dimensionalised and the inverse cascade is balanced by viscosity. Furthermore, close to the transition $S \approx S_c$ we observe complex bi-stable and hysteretical behaviour close and follow the hysteresis curve of the system.

Références

1. Smith, Leslie M., Jeffrey R. Chasnov, and Fabian Waleffe. "Crossover from two-to three-dimensional turbulence." Physical Review Letters 77.12 (1996) : 2467.
2. Benavides, Santiago J., Alexakis, Alexandros "Critical Transitions in Thin Layer Turbulence" J. Fluid Mech. 822, 364 (2017)
3. Stefano Musacchio, Guido Boffetta. "Split energy cascade in turbulent thin fluid layers" Physics of Fluids 29, 111106 (2017)
4. Celani, A., Musacchio, S., Vincenzi, D. "Turbulence in More than Two and Less than Three Dimensions". Physical Review Letters 104, 184506 (2010).
5. Deusebio, Enrico, et al. "Dimensional transition in rotating turbulence." Physical Review E 90.2 (2014) : 023005.
6. Frishman, Anna, and Corentin Herbert. "Turbulence statistics in a 2D vortex condensate." arXiv preprint arXiv :1711.05536 (2017).