Forced three-wave interactions of capillary-gravity surface waves.

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Resonant three-wave interactions

- Generation of a daughter 3 wave from the interaction of two mother waves 1 and 2 by the three wave resonant mechanism.

**Resonant conditions:**

\[ k_1 + k_2 = k_3 \]

\[ \omega_1(k_1) + \omega_2(k_2) = \omega_3(k_3) \]

with \( \omega(k) = \sqrt{gk + \left(\frac{\gamma}{\rho}\right)k^3} \)

\[ \cos(\alpha_{12r}) = \frac{k_3^2 - (k_1^2 + k_2^2)}{2k_1k_2} \]

Selection of a resonant angle

\( F_1 = 15 \text{ Hz} \) et \( F_2 = 18 \text{ Hz} \) \( \Rightarrow F_3 = 33 \text{ Hz} \) and \( \alpha_{12r} = 54^\circ \)


- Now, when \( \alpha \neq \alpha_{12r} \) we report generation of a daughter wave, verifying the resonant conditions, but not the dispersion relation.
Interpretation as a forced three-wave interaction.

- Weakly non-linear model, describing the forcing of the wave $3$ ($f_3, k_3$) by the product of the two mother waves $1$ and $2$.

Resonance when $\alpha = \alpha_{12r}$: forcing matches the dispersion relation.

Like for a forced oscillator, the bandwidth of the resonance is increased by the dissipation.

- When $\alpha \neq \alpha_{12r}$, the daughter wave is spatially modulated. Strong analogy with the case of non-resonant interactions.

- Significant dissipation for capillary waves suggests that forced and non resonant three-wave interactions should be taken into account in the theoretical description of capillary wave turbulence.