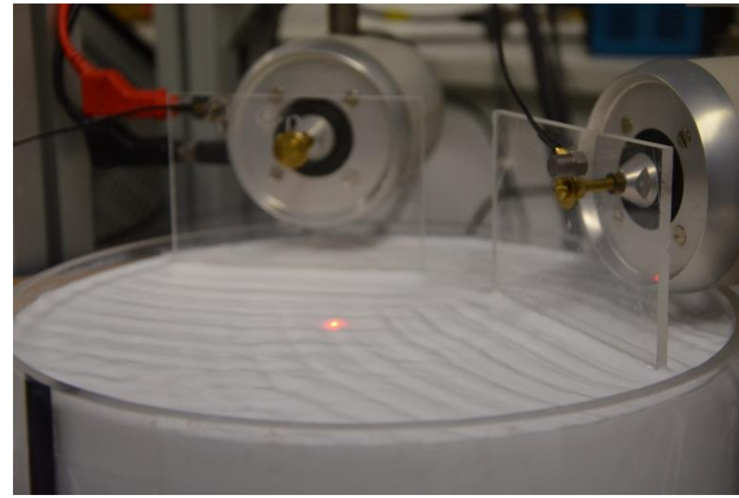
Forced three-wave interactions of capillary-gravity surface waves.

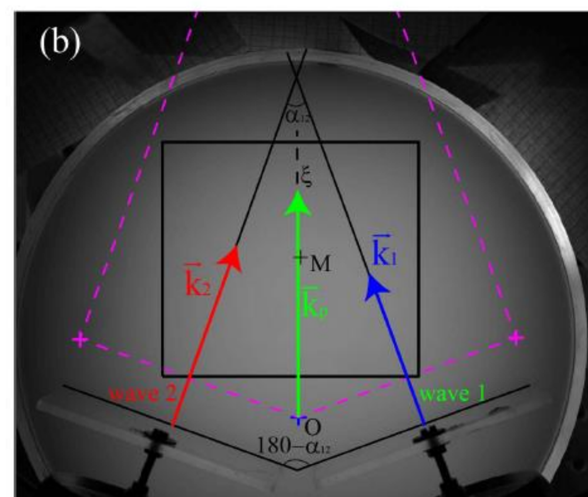
MSC: Laboratoire Matière et Systèmes Complexes
UMR 7057 CNRS
Université Paris Diderot

Michael Berhanu (CR CNRS)

with

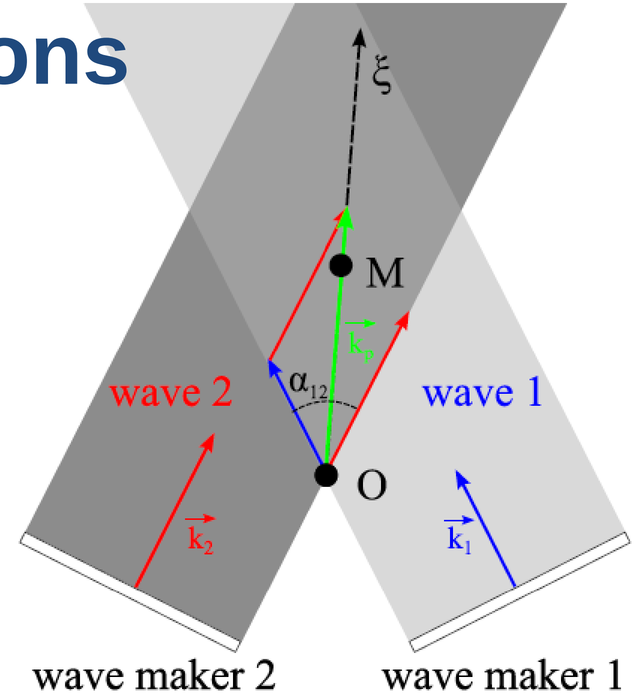
Annette Cazaubiel (Phd Student)
Florence Haudin
(Former Postdoctoral Researcher)
& Eric Falcon (DR CNRS)





Acknowledgments:
Luc Deike (Assistant Professor, Princeton USA)

Resonant three-wave interactions



- Generation of a daughter 3 wave from the interaction of two mother waves 1 and 2 by the three wave resonant mechanism.

Resonant conditions:

$$\mathbf{k}_1 + \mathbf{k}_2 = \mathbf{k}_3 \qquad \omega_1(\mathbf{k}_1) + \omega_2(\mathbf{k}_2) = \omega_3(\mathbf{k}_3)$$

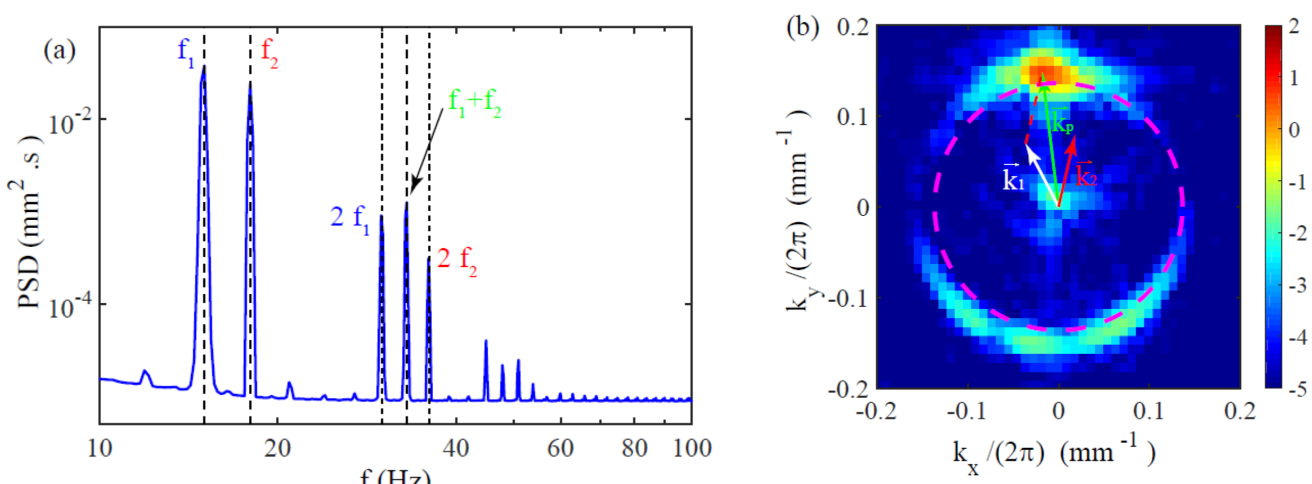
with $\omega(k) = \sqrt{gk + (\gamma/\rho)k^3}$ $\quad \cos(\alpha_{1\,2\,r}) = \dfrac{k_3^2 - (k_1^2 + k_2^2)}{2\,k_1.k_2}$

Selection of a resonant angle

F_1=15 Hz et F_2=18 Hz $\Rightarrow$ F_3=33 Hz and α_{12r} = 54°

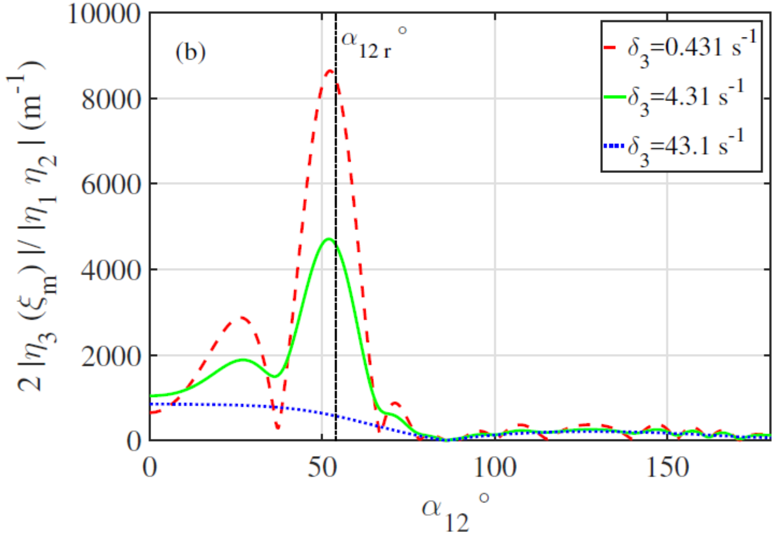
Previous work: **Haudin, Cazaubiel, Deike, Jamin, Falcon & Berhanu, Phys. Rev. E 93, 043110 (2016):** Experimental study of three-wave interactions among capillary-gravity surface waves.

- Now, when $\alpha \neq \alpha_{12r}$ we report generation of a daughter wave, verifying the resonant conditions, but not the dispersion relation.



Interpretation as a forced three-wave interaction.

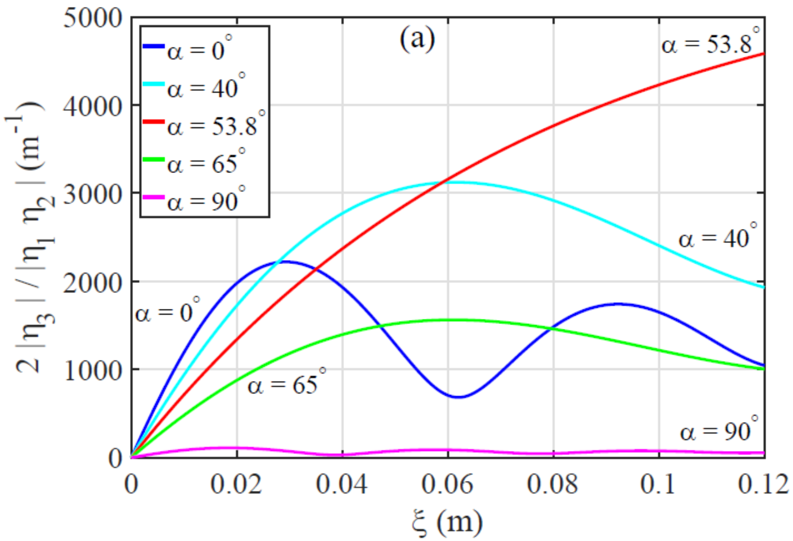
● Weakly non-linear model, describing the forcing of the wave 3 (f_3, k_3) by the product of the two mother waves 1 and 2.



Resonance when $\alpha = \alpha_{12r}$:
forcing matches the dispersion relation.

Like for a forced oscillator, the bandwith of the resonance is increased by the dissipation.

● When $\alpha \neq \alpha_{12r}$,
the daughter wave is spatially modulated. Strong analogy with the case of non-resonant interactions.



● Significant dissipation for capillary waves suggests that forced and non resonant three-wave interactions should be taken into account in the theoretical description of capillary wave turbulence.