Perturbation Theory in Algebraic Setting

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We propose a perturbation algorithm which works on any Lie algebra $\mathbb{V}$. We consider a dynamical system on $\mathbb{V}$, which preserves a subalgebra $\mathbb{B}$ of $\mathbb{V}$. In general when we add a perturbing term to the original dynamical system, the subalgebra $\mathbb{B}$ won't be preserved anymore. We show that under suitable hypothesis, there exists a new dynamical system, conjugated to the perturbed one, which preserves $\mathbb{B}$ up to terms quadratic in the perturbation. By this formula, classical perturbation theory can be extended to noncanonical Poisson system : for instance, the Euler-Poinsot equations for a rigid body, ideal MHD, the Maxwell-Vlasov and Vlasov-Poisson systems. In particular, we show that, for a time dependent top, it is possible to iterate the formula and to get a KAM theorem : for a large set of initial data, the algebra $\mathbb{B}$, like the tori of classical mechanics, get deformed into a new algebra preserved by the perturbed flow.