Rare transitions to a thin-layer turbulent condensate

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A turbulent flow in a thin layer spontaneously develops an inverse cascade of energy and a spectral condensate of energy when the layer height is smaller than a threshold, \cite{Smith1996} & \cite{Celani2010}. Recently, evidence for bistability was found in this system close to the critical height: for the same value of the parameters and depending on the initial conditions the flow is either in a condensate state with most of the energy in the two-dimensional large scale modes or it is in a three-dimensional turbulent state with most of the energy in the small scale modes \cite{vanKan2019}. Bistable behaviour has been identified in a number of turbulent flows, such as fast rotating convection \cite{Favier2019} and von Kármán flows \cite{Ravelet2004}. The presence of noise induces random transitions between the two locally stable states whose statistics characterise the bistable regime.

Here, we report our results on the statistical properties of the thin-layer flow in this bistable regime for both stochastic and deterministic forcing inferred from a large number of direct numerical simulations and measuring the decay time \(\tau_d\) of the condensate to a three-dimensional and the build-up time \(\tau_b\) of the condensate. It is shown that both of these times \(\tau_d, \tau_b\) display an exponential distribution with their mean value increasing (and possibly diverging) close to the threshold.

We further show that the dynamics of large-scale kinetic energy \(E_2\) may be approximated by solving a Langevin equation with multiplicative noise,

\[
\partial_t E_2 = -\frac{\partial U(E_2)}{\partial E_2} + \sqrt{2D(E_2)}E_2.
\]

A transition through a bistable regime as a function of layer height is clearly identified via a transition of \(U(E_2)\) from a single minimum to two minima back to one minimum. Small memory effects are identified via non-Gaussian tails of the conditional transition probability.

Références