Critical transition in fast-rotating turbulence within highly elongated domains

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Rotating fluid flows are commonly encountered in astrophysical and geophysical systems such as planetary and stellar interiors, planetary atmospheres and oceans [1], as well as in industrial processes involving rotating machinery. The fluid motions in these systems are typically turbulent, i.e. the Reynolds number Re, which is defined as the ratio between inertial and viscous forces, is large. At the same time the flow is affected by the Coriolis force due to system rotation. The magnitude of the Coriolis acceleration compared to the inertial acceleration is measured by the non-dimensional Rossby number $Ro = U/(\Omega L)$, where is the rotation rate and U and L are typical velocity and length scales of the flow.
As is well known, the properties of turbulent cascades strongly depend on the number of spatial dimensions. In homogeneous isotropic 3-D turbulence, energy injected at large scales is transferred, by non-linear interactions, to small scales in adirect energy cascade. In the two-dimensional (2-D) Navier-Stokes equations both energy and enstrophy are inviscid invariants and this fact constrains the energy transfer to be from small to largescales in an inverse energy cascade. For 3-D turbulent flow forced at a length scale ℓ in a domain of height $H = h\ell$, when Ro is lowered below a certain threshold value $Ro_c(h)$ in a rotating turbulent flow, a transition is encountered where the flow becomes quasi-2-D and an inverse cascade develops [2]. The dependence of Ro_c on h in the limit of elongated domains and rapid rotation is difficult to investigate using brute force numerical simulations.
Here [3], we use an asymptotic expansion due to [4], valid at simultaneously large rotation rate $\Omega = O(1/\epsilon)$ and domain height $H = O(1/\epsilon)$, $\epsilon \ll 1$, to study such rapidly-rotating turbulent flows in a highly elongated domain. We solve the resulting equations using an extensive set of direct numerical simulations for different parameter regimes. As the parameter $\lambda = (hRo)^{-1}$ is increased beyond a threshold λ_c, a transition is observed from a state without an inverse energy cascade to a state with an inverse energy cascade. This is in agreement with previous theoretical arguments [5]. For large Reynolds number and large horizontal box size, we provide evidence for criticality of the transition in terms of the large-scale energy dissipation rate and estimate a critical exponent close to unity.

Références

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