Parametric wrinkling instabilities of $1D$ structures in spatially periodic elastic states

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Parametric instabilities, that can appear in linear time-periodic systems, are the consequences of the eventual frequency lock-in of the Floquet Forms (FFs) of a linear Ordinary Differential Equation with periodic coefficient [1]. They are sometimes exploited to perform interesting functionalities in structural dynamics such as amplifiers [2] or large-band energy harvester in Micro Electromechanical Systems [3]. One strong limitation though, in fully exploiting parametric instabilities in dynamical systems governed by Initial Value Problems, is that they rapidly disappear with inherent friction forces. Here, we investigate what would be the analog of FFs lock-in in Boundary Value Problems and whether it could be interesting for the stability of structures. We illustrate, throughout the academic problem of a compressed beam on a periodically varying Winkler foundation [4], the mechanics of wavenumber lock-in of Floquet forms and highlight with a more practical structural stability problem, how it could be exploited to enrich the spectrum of elastic buckling patterns.

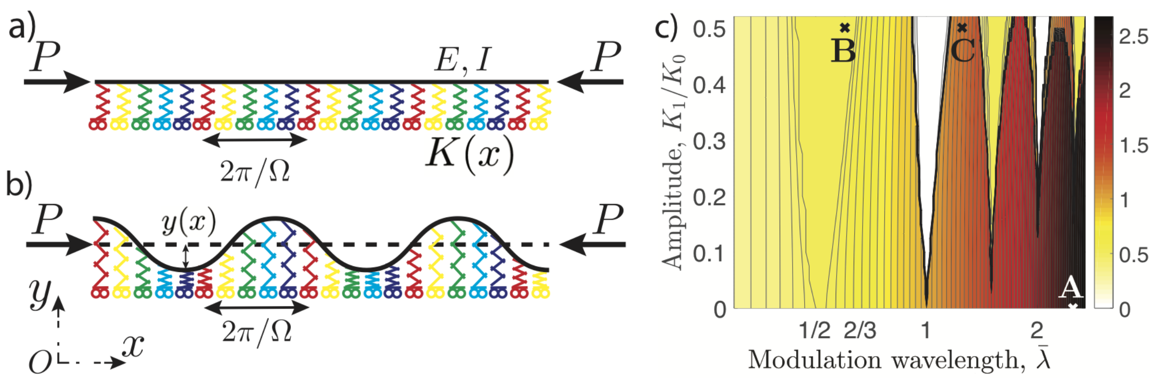

Figure 1. a)-b) Sketch of an infinite compressed beam lying on a Winkler foundation with elastic springs that are harmonically varying in space. The harmonic modulation is characterized by an amplitude K_1 and a wavelength λ. a) Undeformed and b) deformed configuration. c) Evolution of the fundamental wavelength of the buckling pattern as a function of modulation parameters λ and K_1. For certain regions, analogous to Mathieu's tongues, the buckling pattern is not quasi-periodic but 2λ (yellow) or λ-periodic (white).

Références

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