Playing with a rope on the ground: a toy-model for dynamical elastic contacts.

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We study theoretically, numerically, and experimentally the dynamics of a rope laid on the ground when one end is lifted and harmonically vibrated vertically with a small amplitude around a given value. Intuitively, at small frequencies, only the part of the rope that is lifted should be set into motion, the rest of it remaining inert. Meanwhile, the location of the first contact with the ground, $x_c(t)$, should advance and recede, following the up and down motion at the lifted end of the rope. Next, a critical frequency is expected, above which waves are radiated along the rope away from the vibrated end. Our study confirms this intuition and yields the critical frequency of this transition. In the limit of small exictations $0 < \epsilon \ll 1$, we find that

$$x_c \sim x_{c,0}\left[1+\frac{\epsilon}{2}\frac{\pi\omega/\omega_c}{\sin(\pi\omega/\omega_c)}\cos(\omega t)+O\left(\epsilon^2\right)\right], \qquad \omega_c=\left(\frac{\pi^2 g}{2Z_0}\right)^{1/2}$$

The above result indicates that the response diverges as $\omega \to \omega_c$. As one approaches the resonant pulsation ω_c, the amplitude of vibration of the rope increases to such an extent that the rope can enter in contact with the ground at an intermediate location x^* which leads to a propagating bump as expected. Thus, a dynamical transition is observed between a state of localised oscillations and a state of wave radiation along the rope. This dynamical scenario is confirmed both experimentally and by numerical simulations using Moreau's algorithm.

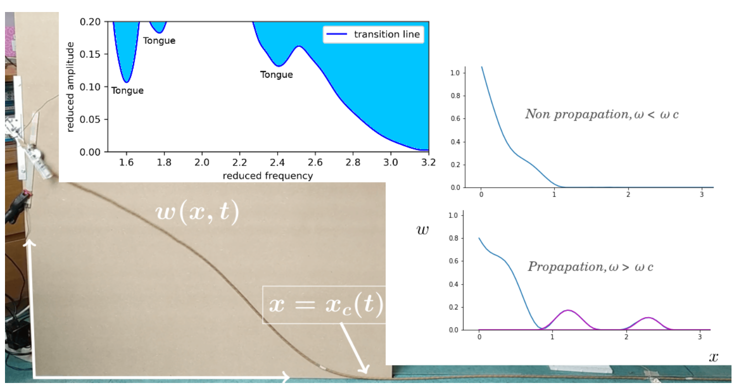

Figure 1. The dynamic experiment (left) and simulations (right), where one can observe both regimes, above and below the critical frequency.

The full story, however, is far richer than anticipated. The contact point x_c behaves very erraticaly, having an infinite number of resonances $\omega_n = n\omega_c$. This infinite set of resonances indicates that the rope is a more complicated dynamical object than a simple pendulum: They are the signature of delayed interactions mediated by waves travelling up and down the rope.

Even more interestingly we showed that an infinity of secondary critical frequencies exist below ω_c and that the stability diagram is actually covered by Arnold's tongues. This runs against the intuition of a simple mechanical system with a single critical frequency. The richness of Arnold's tongues has been checked numerically with great accuracy.