

# On-off intermittency due to parametric Lévy noise

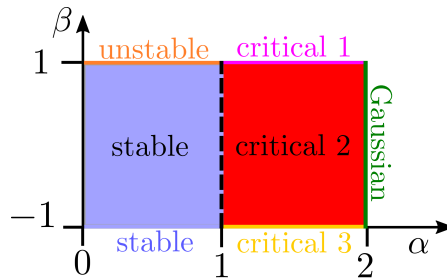
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Instabilities arise in many physical systems at some parameter threshold. Typically the system is embedded in an uncontrolled noisy environment. The fluctuating properties of the environment affect the control parameters of the instability, which leads to multiplicative noise. The result of multiplicative noise close to an instability threshold is on-off intermittency, which is characterised by an aperiodic switching between a large-amplitude “on” state and a small-amplitude “off” state.

Here, we present a new type of intermittency, *Lévy on-off intermittency*, which arises from multiplicative  $\alpha$ -stable white noise close to an instability threshold [1]. We study this problem in the linear and nonlinear regimes, both theoretically and numerically, for the case of a pitchfork bifurcation with fluctuating growth rate. In a recently introduced point-vortex model of 3-D perturbations in 2-D flows [2], the perturbation amplitude obeyed such an equation. We compute the stationary distribution analytically and numerically from the associated fractional Fokker-Planck equation in the Stratonovich interpretation. We characterize the system in the parameter space  $(\alpha, \beta)$  of the noise, with stability parameter  $\alpha \in (0, 2)$  and skewness parameter  $\beta \in [-1, 1]$ . Five regimes are identified in this parameter space, in addition to the well-studied Gaussian case  $\alpha = 2$ . Three regimes are located at  $1 < \alpha < 2$ , where the noise has finite mean but infinite variance. They are differentiated by  $\beta$  and all display a critical transition at the deterministic instability threshold, with on-off intermittency close to onset. Critical exponents are computed from the stationary distribution. Each regime is characterised by a specific form of the density and specific critical exponents, which differ starkly from the Gaussian case. A finite or infinite number of moments may converge, depending on parameters. Two more regimes are found at  $0 < \alpha \leq 1$ . There, the mean of the noise diverges, and no transition occurs. In one case the origin is always unstable, independently of the distance  $\mu$  from the deterministic threshold. In the other case, the origin is conversely always stable, independently of  $\mu$ . Figure 1 shows the resulting parameter space. In summary, we demonstrate that an instability subject to non-equilibrium, power-law distributed fluctuations can display substantially different properties than for Gaussian thermal fluctuations, in terms of statistics and critical behavior.



**Figure 1.** State diagram of a pitchfork bifurcation whose growth rate exhibits Lévy white noise fluctuations.

## Références

1. A. van Kan, A. Alexakis & M. E. Brachet, Lévy on-off intermittency, arXiv :2102.08832
2. A. van Kan, A. Alexakis & M. E. Brachet, Intermittency of three-dimensional perturbations in a point-vortex model, arXiv :2012.12637