Unstable frameworks, symmetry groups and quaternions

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Frameworks of bars have been long used in architecture and civil engineering for producing light and resistant structures. Due to their highly symmetric geometries, these structures, when submitted to compressive forces, can undergo intricate unstable behaviors, one very famous example of them being the hexadome structure, first presented by [1]. Using group-theory, [1] studied a very flexible version of the hexadome where all the rods were pin-balled with one another, thus leading to only tension or compression in the bars. In this work, we will study the more realistic problem of fixed boundary conditions with the

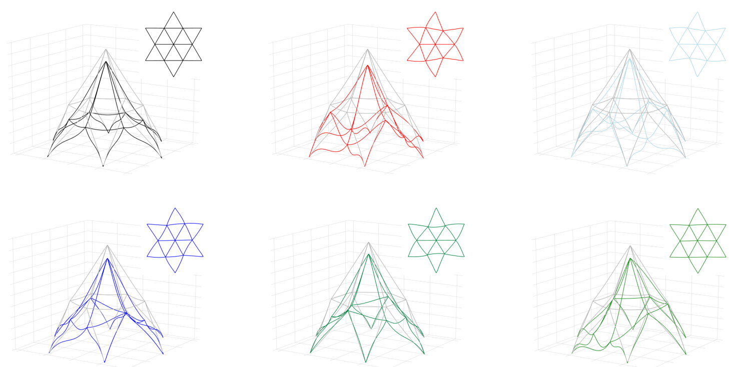

Figure 1. Deformed configurations (elevation and top views) for the hexadome with fixed boundary conditions

ground and between the bars along with various cross-sections, thus leading to the possibility of flexion or torsion in the bars. This complex study is eased by the use of group symmetry [3] for the study of the possible unstable branches and quaternions [5,4] to model possible large rotations of the bars. A force-based model [2] taking advantage of the conservation law for the unit quaternion constraint is used in the bars. It will be shown that quaternions can be used in the energy invariance proof necessary to allow us to use symmetry group theory. Details on the point-group symmetry of the structure and its decomposition into subgroups will be given, the deformed configurations (Fig.1) and the bifurcation diagram presented.

Références

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