Wave Turbulence in Self-Gravitating Bose Gases and Nonlocal Nonlinear Optics

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We develop the theory of weak wave turbulence in systems described by the Schrödinger-Helmholtz equations in two and three dimensions. This model contains as limits both the familiar cubic nonlinear Schrödinger equation, and the Schrödinger-Newton equations. The latter, in three dimensions, are a nonrelativistic model of fuzzy dark matter which has a nonlocal gravitational self-potential, and in two dimensions they describe nonlocal nonlinear optics in the paraxial approximation. We show that in the weakly nonlinear limit the Schrödinger-Helmholtz equations have a simultaneous inverse cascade of particles and a forward cascade of energy. The inverse cascade we interpret as a nonequilibrium condensation process, which is a precursor to structure formation at large scales (for example the formation of galactic dark matter haloes or optical solitons). We show that for the Schrödinger-Newton equations in two and three dimensions, and in the two-dimensional nonlinear Schrödinger equation, the particle and energy fluxes are carried by small deviations from thermodynamic distributions, rather than the Kolmogorov-Zakharov cascades that are familiar in wave turbulence. We develop a differential approximation model to characterise such "warm cascade" states.