Explicit analysis of catasrophe on a Timoshenko beam

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Analytical solutions of a plane, quasi-static but large transformation of a Timoshenko beam is exposed. The problem is first re-formulated in the form of a Cauchy initial value problem where load (force and moment) is prescribed at one-end and kinematics (translation, rotation) at another. With such formalism, solutions are explicit for any load. The existence, uniqueness and regularity of the solution of the problem are proven. Therefore, analytical post-buckling solutions were found with different regimes driven explicitly by two invariants of the problem. Pure shear follower load is examined where explicit solutions are obtained in terms of kinematical and dynamical variables. Qualitative and quantitative analyses are illustrated.
Boundary value problem is presented, where physical quantities (of load, position or section orientation) are prescribed at both ends. Explicit solution is presented in the case of dead load ($\boldsymbol{N}_\ell$) at one end whereas orientation of the section is controlled at the other. In this case brutal change of the configuration of the beam is observed which leads to catastrophic instability. Here, an explicit solution is a way to anticipate such catasrophe as it is the occasion to predict and control the parameters driving this phenomena.

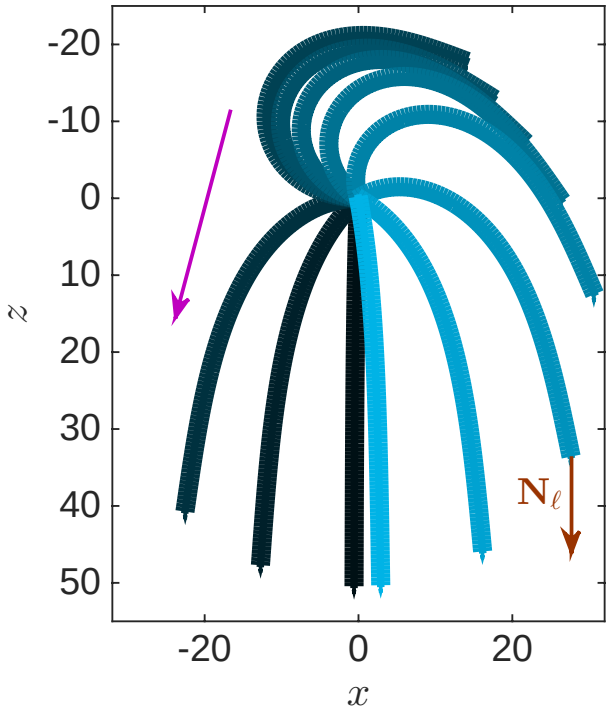

Figure 1. Successive configurations for various values of θ_0 respecting a boundary condition command. The purple arrow highlights the brutal transition of the configurations during the catastrophe.