Parametric wrinkling instabilities of 1D structures in spatially periodic elastic states

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Parametric instabilities (PIs), that can appear in dynamical systems, are due to internal synchronizations eventually arising in linear time-periodic systems [1]. They are sometimes exploited to perform interesting functionalities in structural dynamics such as amplifiers [2] or large-band energy harvester in Micro Electromechanical Systems [3]. One strong limitation though, in fully exploiting PIs in dynamical systems governed by Initial Value Problems, is that they rapidly disappear with inherent friction forces. Here, we investigate what would be the analog of PIs in Boundary Value Problems and whether it could be interesting for the stability of structures. We illustrate, throughout the academic problem of a compressed beam on a periodically varying Winkler foundation [4], the mechanics of "parametric wrinkling" and highlight with a more practical structural stability problem that is the transverse buckling of an elastic wall with periodically varying height, how it could be exploited to enrich the spectrum of elastic buckling patterns.

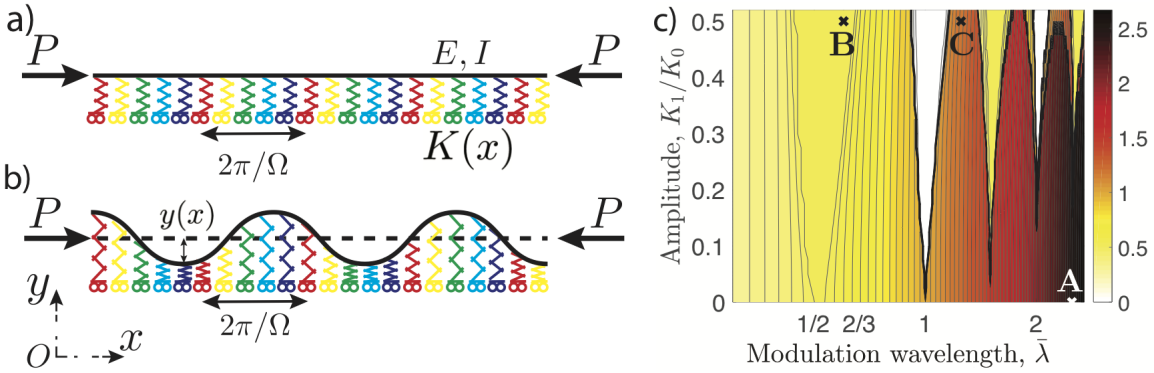

Figure 1. a)-b) Sketch of an infinite compressed beam lying on a Winkler foundation with elastic springs that are harmonically varying in space. The harmonic modulation is characterized by an amplitude K_1 and a wavelength λ. a) Undeformed and b) deformed configuration. c) Evolution of the fundamental wavelength of the buckling pattern as a function of modulation parameters λ and K_1. For certain regions, analogous to Mathieu's tongues, the buckling pattern is not quasi-periodic but 2λ (yellow) or λ-periodic (white).

Références

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