Confined vs. Unconfined: Triadic Resonant Instability in Cylindrical Geometry

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The various non-linear mechanisms through which inertia-gravity waves can be excited at different frequencies and different wave numbers trigger an on-going interest since they are associated to energy transfers from scales to scales and may have a significant role in mixing processes, for example in the oceans. Among these non-linear interactions is the spontaneous generation of sub-harmonics forming resonant a triad with the primary wave field, called Triadic Resonant Interaction (TRI).

Numerous experimental, numerical, and theoretical studies have been conducted to investigate TRI and sub-harmonics in 2D Cartesian geometry, using plane wave formalism (see, e.g. [3]). The triads constituted of the forced wave and the two spontaneously excited sub-harmonics are then found to satisfy a linear resonance condition, both on the frequencies and on the wave numbers, that can be written as

$$\omega_0 = \pm\omega_1 \pm \omega_2,$$
$$\mathbf{k_0} = \pm\mathbf{k_1} \pm \mathbf{k_2},$$

where ω_0 is the frequency of the primary wave and $\mathbf{k_0}$ its wave vector (respectively ω_1, $\mathbf{k_1}$ and ω_2, $\mathbf{k_2}$ for the sub-harmonics). With axisymmetric internal waves in cylindrical geometry, however, the description of the wave field in terms of Bessel functions yields more complicated equations and it is not obvious that the same TRI conditions are satisfied [5]. While evidence of TRI in such geometries have been reported recently [5,6], no clear theory has been derived to characterise the corresponding resonance conditions.

In an unconfined 3D configuration, we derive the relevant resonance conditions cylindrical wave fields satisfy and we show that they are similar to the TRI ones for Cartesian plane waves, although the radial resonance is only approximate. We also show that, when the waves are fully confined, boundary condition prevail, leading to a triad of waves that are non-necessarily spatially resonant. Using an apparatus that has been proven capable of generating axisymmetric internal wave fields [1], we present experimental observations of axisymmetric internal wave sub-harmonics generated in unconfined and confined domains [2], agreeing well with the proposed theory.

Références

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