Direct and inverse cascades in turbulent Bose-Einstein condensate

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Cascades of energy or/and other invariants play the central role in the hydrodynamic and wave turbulence. Recently, WT was implemented experimentally in Bose-Einstein condensates (BEC) [1]. BEC has a great potential as a platform for experiments in turbulence, both of vortex and wave kinds. This is because of the close analogy between the BEC motion and the classical fluid flow, as well as because the BEC experiments allow a great deal of flexibility often unavailable in the classical fluid experiments. Moreover, the Gross-Pitaevskii equation (GPE) describing the BEC dynamics appears to be a universal nonlinear model whose importance spans over diverse physical systems, in particular in optics, plasmas and water wave theory [2]. The wave turbulence theory (WWT) is a mathematical framework describing the statistical behavior of WT dominated by weakly nonlinear waves [3]. The main object in this theory is the wave-action spectrum which is the second-order moment of the wave amplitude and which evolves according to the so-called wave-kinetic equation (WKE). The most remarkable achievement of WWT is that it gives the stationary scaling solutions by solving WKE analytically. The power-law scaling solutions in WWT are called Kolmogorov–Zakharov (KZ) spectra, which are analogous of Kolmogorov spectra in hydrodynamic turbulence. The latter, on the other hand, can be derived only by dimensional analysis.

Let us focus on WT in the four-wave regime of the 3-dimensional Gross-pitaevkii system, where the effect of condensate is negligible. The associated WKE predicts two non-equilibrium stationary power laws: k^{-1} of the spherically-integrated wave-action spectrum for the direct cascade of energy, and $k^{-1/3}$ for the inverse cascade of particles. To obtain the stationary solutions given by WWT, we numerically solved the GPE and WKE respectively at high resolutions with proper external forcing injected and dissipation at the largest and smallest scales.

It is notable that the recent WT experiments in BEC implemented by [1] reported a mysterious scaling of $k^{-1.5}$ for the direct cascade. To fix this, we applied a log-correction to the power-law solution, similar as in 2-dimensional hydrodynamic turbulence. The numerical results of both GPE and WKE, it turns out, match the log-corrected KZ spectrum quite well. In hydrodynamic turbulence theory, it is well known that the flux across the inertial range determines the power-law energy spectrum. As previously stated, the dimensional analysis provides this relationship with Kolmogorov spectrum, but leaves an arbitrary constant to be fitted by experiments or numerical simulations. Fortunately, in WWT, it is possible to derive the analytical expression for such a similar constant and to compute its value. In this work, we found the universal constant of the KZ spectrum for the inverse cascade. In addition, the $k^{-1/3}$ law is achieved for the first time in numerical simulations. The wave-action spectra generated by GPE and WKE correspond very well with the prediction by the flux of particles using the constant we obtained.

References

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