Modeling and numerical simulation of elastic turbulence in polymer solutions

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Elastic turbulence: A chaotic flow that emerges in polymer solutions at low Reynolds number and high Weissenberg number

Modeling and numerical simulation of elastic turbulence in polymer solutions

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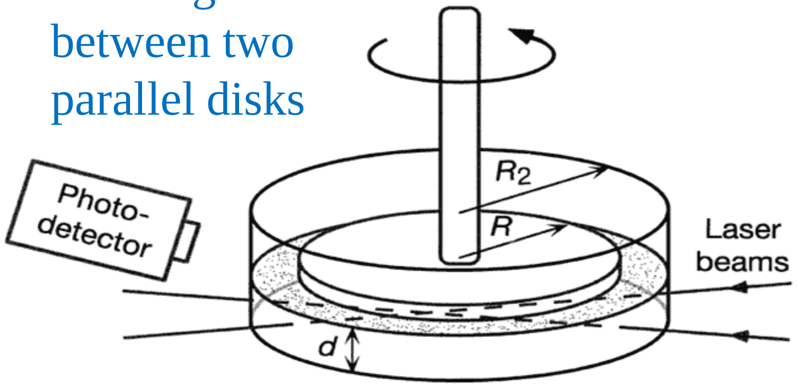
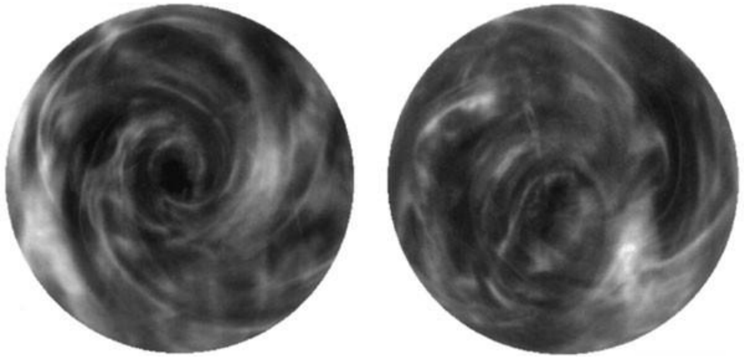
Swirling flow between two parallel disks



Irregular patterns and spiral-like structures



Wi=13, Re=0.7

A. Groisman & V. Steinberg, Nature, 2000

V. Steinberg, Annu. Rev. Fluid Mech., 2021

Governing equations for the viscoelastic fluids

$$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \boldsymbol{\nabla u} = -\boldsymbol{\nabla} p + \nu \boldsymbol{\nabla}^2 \boldsymbol{u} + \frac{\mu}{\tau} \boldsymbol{\nabla} \cdot \boldsymbol{C} + \boldsymbol{f}$$ – Velocity field

$$\frac{\partial \boldsymbol{C}}{\partial t} + \boldsymbol{u} \cdot \boldsymbol{\nabla C} = \boldsymbol{C} \cdot \boldsymbol{\nabla u} + (\boldsymbol{\nabla u})^T \cdot \boldsymbol{C} - \frac{1}{\tau}(\boldsymbol{C} - \boldsymbol{I})$$ –Conformation tensor field

Preserving positive-definiteness of the conformation tensor is a great challenge in the numerical simulations

Numerical treatments:

- Addition of a diffusive term *i.e.* $\kappa(\boldsymbol{C}, \boldsymbol{\nabla u})\boldsymbol{\nabla}^2 \boldsymbol{C}$
- Use of shock-capturing schemes like *Kurganov-Tadmor scheme*
- Matrix decompositions of the constitutive equations

Two widely used matrix decompositions

- Log Cholesky : $\boldsymbol{C} = \boldsymbol{L}\boldsymbol{L}^T$, $\boldsymbol{L}$ is the lower triangular matrix
- Symmetric square root : $\boldsymbol{C} = \boldsymbol{b}\boldsymbol{b}^T$, $\boldsymbol{b}$ is the symmetric square root of $\boldsymbol{C}$

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- *Are both these decompositions equally accurate?*
- *What are the criteria that an accurate numerical solution must satisfy?*
- *What are the effects of an artificial diffusive term?*

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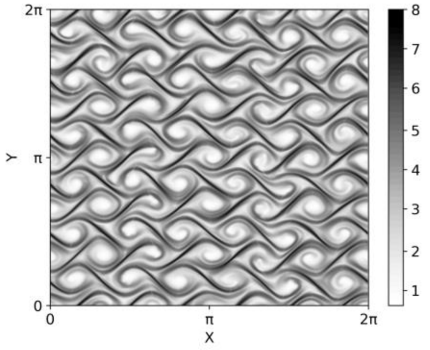
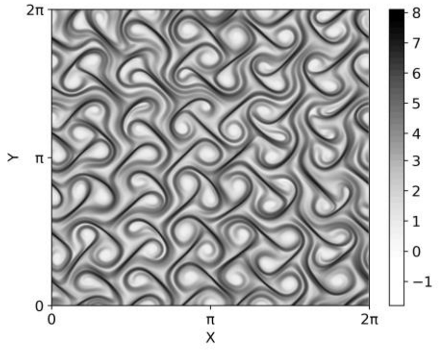
Simulation results:

Log-Cholesky



Different large-scale structures

Symmetric square root



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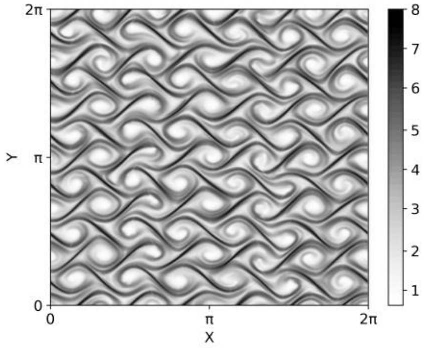

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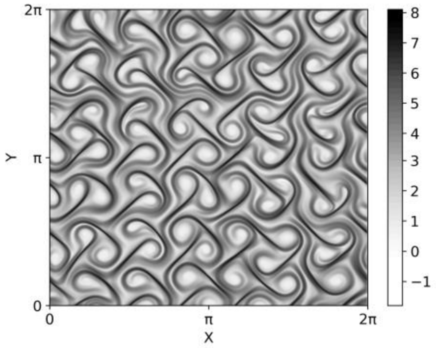
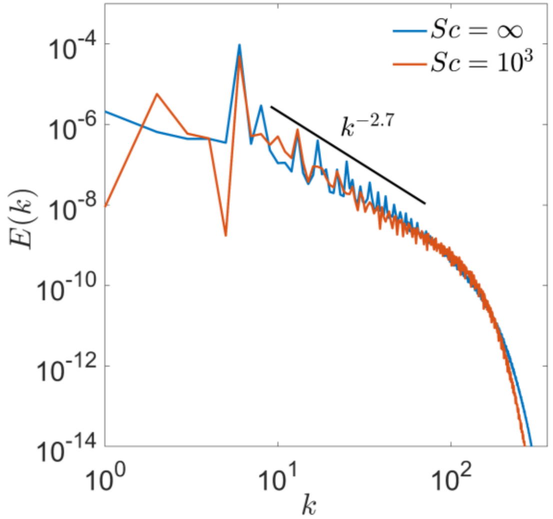


Addition of a diffusive term



An increase and decrease of energy at large scales is observed when a diffusive term is incorporated