

Recent progress on smooth self-similar solutions to the compressible Euler equations

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1 Introduction

2 Setup

3 Acoustic Cone

4 Stability

5 Unstable Singularity

6 Conclusions

7 References

Smooth Self-Similar Solutions & Smooth Perturbations

Main Results [5]

- Numerical construction of SMOOTH self-similar solutions.
- Stability analysis of SMOOTH self-similar solutions.
- Singularity formation in Implosions is UNSTABLE.

Self-Similar Solutions

References

Istropic Compressible Euler Equations

$$\partial_t \rho + \nabla \cdot (\rho u) = 0 \quad (\text{Mass})$$

$$\rho \partial_t u + \rho u \cdot \nabla u + \nabla p = 0 \quad (\text{Momentum})$$

$$p \propto \rho^\gamma \quad (\text{EoS})$$

Singularities

Implosions: $\rho, |u| \rightarrow \infty$

Shocks: $|\nabla \rho|, |\nabla u| \rightarrow \infty, \quad |\rho|, |u| < \infty$

Implosions \Rightarrow Self-Similar Solutions

$$\rho(t, x) = \frac{1}{(T-t)^\alpha} \hat{\rho} \left(\frac{x}{(T-t)^\beta} \right)$$

Smooth Self-Similar Solutions

20th century: Non-Smooth

Guderley [1942]
Taylor [1950]
Sedov [1959]
Lazarus [1984]

21th century: Smooth

Merle et al [2019]
Biasi [2021]
Buckmaster et al [2022]

20th-century

- **Guderley [1942]:** Radially symmetric self-similar solutions: **non-smooth** (e.g. shocks).
- **Lazarus [1984]:** Stability of **non-smooth** Guderley solutions

21st-century

- **Merle-Raphael-Rodnianski-Szeftel [2019]:** Existence of **smooth** Guderley solutions.
- **Biasi [2021]:**
 - New **smooth** Guderley solutions (numerical).
 - Stability **smooth** Guderley solutions.
 - Instability of implosions (shock formation).
- **Buckmaster, Cao Labora, Gómez Serrano [2022]:** New **smooth** Guderley solutions (rigorous).