Four compartment epidemic random walk model

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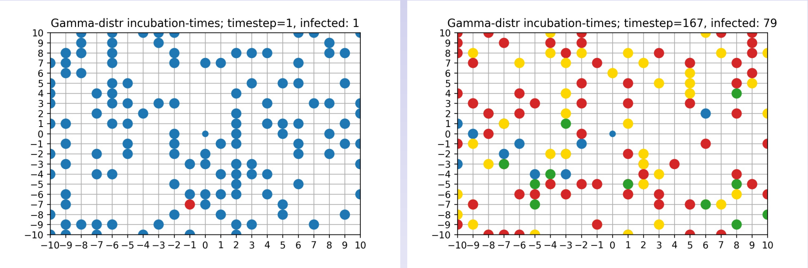

Figure: Colors of indicate health status S C I R of the walkers.

• $Z \gg 1$ random walkers navigate independently on a $N \times N$ square-lattice, jumping with probability $1/4$ to any of four neighbor lattice points

• Each walker is in one of the states ('compartments')

S : susceptible (for infection)
C : incubated, infected but not infectious
I : infected and infectious
R : recovered and immune

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Four compartment SCIRS model

Infection rule :

If: S meets I on the same lattice point – collision of I and S walkers

Then: the S walker gets infected with probability P_{inf}

performing transition

S $\rightarrow$ C with probability P_{inf}

followed by delayed transition C $\rightarrow$ I $\rightarrow$ R $\rightarrow$ S

with random sojourn times t_C, t_I, t_R in the compartments

drawn from probability density functions

$$\mathbb{P}(t_{C,I,R} \in [\tau, \tau + \mathrm{d}\tau]) = K_{C,I,R}(\tau)\mathrm{d}\tau$$

Four compartment SCIRS model

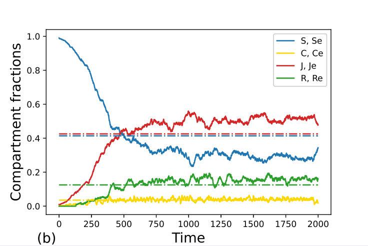
Evolution equations $s(t)+c(t)+j(t)+r(t)=1$ (constant population without deaths)

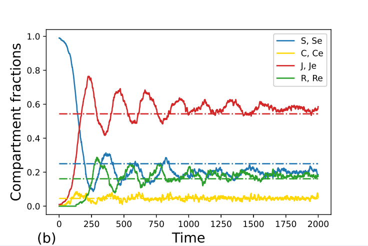
$$\begin{aligned}
\frac{d}{dt}s(t) &= -\mathcal{A}(t) + (\mathcal{A}\star K_C \star K_I \star K_R)(t) \\
\frac{d}{dt}c(t) &= \mathcal{A}(t) - (\mathcal{A}\star K_C)(t) \\
\frac{d}{dt}j(t) &= (\mathcal{A}\star K_C)(t) - (\mathcal{A}\star K_C \star K_I)(t) \\
\frac{d}{dt}r(t) &= (\mathcal{A}\star K_C \star K_I)(t) - (\mathcal{A}\star K_C \star K_I \star K_R)(t)
\end{aligned}$$

$(a\star b)(t) = \int_0^t a(\tau)b(t-\tau)\mathrm{d}\tau$ stands for convolution
$\mathcal{A}(t)$ infection rate, assumption $\mathcal{A}(t) = \beta j(t)s(t)$ nonlinear function of $j(t)$ and $s(t)$ describing probability of collision of I and S walkers.

Four compartment SCIRS model with superspreaders (long-range jumpers)

δ-kernels: $K_{C,I,R}(\tau) = \delta(\tau - t_{C,I,R})$, i.e. constant t_C, t_I, t_R for all walkers





Left plot: Epidemic evolution without superspreaders, all walkers perform local jumps.

Right plot: 30% superspreaders = fraction of walkers performing long-range jumps

Simulation with $Z = 100$ walkers, N^2 lattice points ($N = 41$), density of walkers $Z/N^2 \approx 0.06$, infection probability $P_{inf} = 0.9$ with $t_C = 10$, $t_R = 40$ and large time of illness $t_I = 130$ sojourn times. We count at each time increment the fractions $s(t) = \frac{Z_s(t)}{Z}, c(t) = \frac{Z_c(t)}{Z}, j(t) = \frac{Z_I(t)}{Z}, r(t) = \frac{Z_R(t)}{Z}$.

SCIRS model

References:
[1] W.O. Kermack, A.G. McKendrick, A contribution to the mathematical theory of epidemics, Proc. Roy. Soc. A 115, 700–721 (1927)
[2] e T. Granger, T.M. Michelitsch, M. Bestehorn, A. P. Riascos, B. A. Collet, *In Press*, Phys. Rev. E, Preprint: arXiv:2210.09912 (2022).
[3] M. Bestehorn, T. M Michelitsch, B. A. Collet, A. P. Riascos, A. F. Nowakowski, Phys. Rev. E, 105, 024205, (2022).

- For animated simulations and details of the model consult:

https://sites.google.com/view/scirs-model-supplementaries/accueil