Integrable turbulence in the adiabatic pumping regime: from small noise to bound-state soliton gas

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Within the framework of the focusing one-dimensional nonlinear Schrodinger equation (1D-NLSE) supplemented by a small linear pumping term,

$$i\psi_t + \psi_{xx} + |\psi|^2\psi = ip_0\psi, \tag{1}$$

we study numerically the turbulence that develops from statistically homogeneous Gaussian noise of small amplitude. For sufficiently small initial noise and pumping coefficient, and also for not very wide simulation box (basin length), we observe that such turbulence grows in a universal adiabatic regime, moving successively through the statistically stationary states of the integrable 1D-NLSE: as soon as the pumping is turned off, the resulting integrable turbulence [1] turns out to be stationary and does not depend on the pumping coefficient, amplitude of the initial noise or basing length. Focusing on this adiabatic pumping regime, we study how the integrable turbulence grows from small (linear) noise, goes through states of intermediate nonlinearity and reaches strongly nonlinear states characterized by a high frequency of rogue waves. Monitoring this evolution with the IST method, we observe that the solitonic part of the wavefield becomes dominant even when the (linear) dispersion effects are still leading in the dynamics and with increasing average intensity the wavefield approaches a dense bound-state soliton gas, whose properties are defined by the Fourier spectrum of initial noise [2].

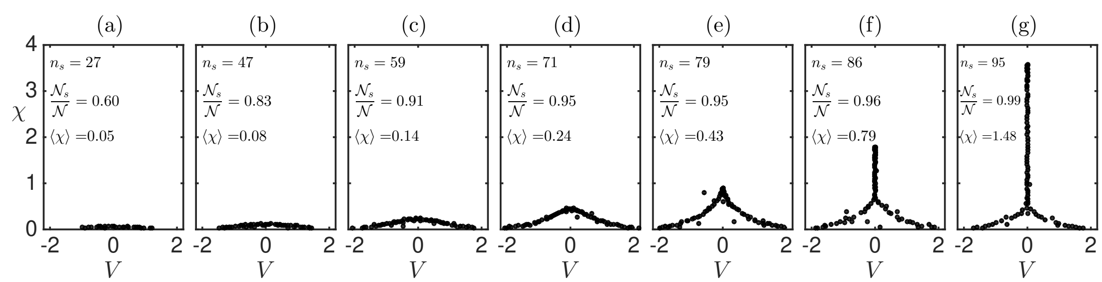

Figure 1. Soliton amplitudes χ and velocities V at different stages of the pumping: (a) $\langle|\psi|^2\rangle^{1/2} = 0.125$, (b) 0.177, (c) 0.25, (d) 0.35, (e) 0.5, (f) 0.71 and (g) 1.0. In the panels, $\mathcal{N}_s/\mathcal{N}$ represents the ratio between the wave action of the solitonic part of the wavefield $\mathcal{N}_s = \langle|\psi_s|^2\rangle$ and the total wave action $\mathcal{N} = \langle|\psi|^2\rangle$.

References

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2. D.S. Agafontsev, A.A. Gelash, R.I. Mullyadzhanov, V.E. Zakharov, Bound-state soliton gas as a limit of adiabatically growing integrable turbulence, *Chaos, Solitons & Fractals*, **166**, 112951 (2023).