Spectral theory of soliton gas in integrable dispersive hydrodynamics

Gennady El

Northumbria University, Newcastle upon Tyne, UK
gennady.el@northumbria.ac.uk

Multi-scale dispersive hydrodynamic flows can exhibit complex spatiotemporal behaviours that require a statistical description even though the underlying physical model can be, in principle, amenable to the well-established mathematical techniques of integrable systems such as the inverse scattering transform and finite-gap theory. At a microscopic scale associated with the system's coherence length, integrable dispersive hydrodynamics feature solitons—the localised nonlinear waves that exhibit particle-like properties such as elastic, pairwise interactions. *Soliton gas* then can be introduced phenomenologically, as an infinite random ensemble of interacting solitons that display a nontrivial collective, hydrodynamic or kinetic behaviour. The theory of soliton gas was initiated by V. Zakharov in 1971 [1], where an approximate kinetic equation for solitons of the Korteweg-de Vries equation was constructed by evaluating the effective adjustment to the tracer soliton velocity in a *rarefied gas* due to its interactions with other solitons, accompanied by the well-defined phase shifts. The generalisation of Zakharov's equation to the case of dense soliton gas has required the development of a new theoretical framework based on the thermodynamic limit of spectral finite-fap solutions to integrable equations and their modulations described by multiphase Whitham theory [2], [3], [4]. Along with novel mathematical aspects related to integrability of the spectral kinetic equation, it has transpired recently that statistics of soliton gas provides important insights in the long-time behaviour of modulationally unstable systems [5]. Additionally, the recently revealed intriguing parallels between kinetic theory of soliton gas and generalised hydrodynamics of integrable many-body systems [6] open an avenue to the construction of soliton gas thermodynamics.

In my talk I will present general ideas of the spectral kinetic theory of soliton gas and outline its perspectives.

References

1. V. E. Zakharov, Kinetic equation for solitons, *Journ. Exp. Theor. Phys.*, **33**, 538, (1971).
2. G. A. El, The thermodynamic limit of the Whitham equations, *Phys. Lett. A*, **311**, 374, (2003).
3. G. A. El and A. Tovbis, Spectral theory of soliton and breather gases for the focusing nonlinear Schrödinger equation, *Phys. Rev. E*, **101**, 052207, (2020).
4. G. A. El, Soliton gas in integrable dispersive hydrodynamics, *Journ. Stat. Mech.*, 114001, (2021).
5. A. Gelash, D. Agafontsev, V. Zakharov, G. El, S. Randoux, and P. Suret, Bound state soliton gas dynamics underlying the spontaneous modulational instability, *Phys. Rev. Lett*, **123**, 234102, (2019).
6. T. Bonnemain, B. Doyon and G.A. El, Generalized hydrodynamics of the KdV soliton gas, *Journ. Phys. A*, **55**, 374004, (2022).