Elastic snap-through instabilities are governed by geometric symmetries

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Snap-through is a general motif for the storage and rapid release of elastic energy. It is exploited by many biological and engineered systems from the Venus flytrap [1] to mechanical metamaterials [2].

Snap-through occurs when a system is in an equilibrium state that becomes unstable or suddenly disappears, as a control parameter is varied. While, these shape transitions are known to be related to the type of bifurcation the system undergoes, to date, there is no general understanding of the mechanisms that select these bifurcations. Here we analyze numerically and analytically two systems proposed in recent literature in which an elastic strip, initially in a buckled state, is driven through shape transitions by either rotating or translating its boundaries. We show that the two systems are mathematically equivalent, and identify three cases that illustrate the entire range of transitions described by previous authors [3,4]. Using reduction order methods, we obtain the normal form of the bifurcations these systems undergo. Importantly, the well known symmetry breaking mechanism that turns a pitchfork into a saddle node bifurcation [5], appears here, in an infinite dimensional system, governing elastic shape transitions. This intuitive yet universal understanding of elastic instabilities based on the symmetries of the system provides powerful tools for the diagnostic and design of elastic structure [6,7].

Références

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