Differential diffusion of bouncing grains

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More than two centuries ago, Chladni noticed that salt grains, when deposited on a vibrated metal plate, spontaneously gather into remarkable geometric figures [1]. We now understand that these figures show the eigenmodes of the vibrating plate—the grains accumulate along nodal lines. The mechanism by which the grains accumulate, however, remains debated. As they move erratically over the plate, one is tempted to see their individual trajectories as random walks, and their collective motion as diffusion. We suggest the grains gather where diffusivity is low, thus forming Chladni figures by differential diffusion.

To test this hypothesis, we design a series of experiments in which the amplitude of the plate's vibration is heterogeneous (figure 1). We observe that the places of lower diffusivity indeed collect more grains. At equilibrium, the surface density of grains, ρ, is inversely proportional to the local diffusivity, D:

$$\rho \propto 1/D. \tag{1}$$

In a diffusivity gradient, random walkers usually generate a macroscopic flux $\mathbf{q}$, whose expression depends on the details of the microscopic dynamics [2]. For jumping grains, we propose

$$\mathbf{q} = -\nabla(D\,\rho) \tag{2}$$

which, in equilibrium, yields equation (1). Particles that diffuse according to this law can also produce non-equilibrium steady states, wherein macroscopic currents stir the grains around the box that contains them [3,4]. We pursue these elusive configurations in our experiments.

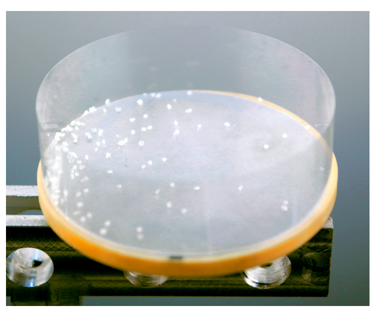

Figure 1. Salt grains bouncing over a vibrated rigid disk (radius 9 cm). The vibration amplitude increases from left to right.

References

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