Multi-scale turbulent synthesis

Pierre Lesaffre[1]

LPENS, 24 rue Lhomond 75231 Paris Cedex
lesaffre.pierre@gmail.com

Give a gentle push to a puff of smoke : it soon develops convoluted motions, diverging trajectories blur the initial impulse, but nevertheless some distinctive features remain. In this apparent chaos, the human eye is expert at recognising waves, spiraling structures, vortices, shocks, eddies... Also, hierarchical order emerges from this apparent randomness : small structures are embedded in bigger ones, which manifest in the celebrated Kolmogorov (1941) spectral power law. Turbulence studies look for a number of generic statistical properties shared by flows after they have lost the imprint of their initial conditions : structure functions, statistics of increments, multifractal spectra, etc...

Today, simulations can predict the evolution of fluid motions from random initial conditions and we can try to assess what statistical laws control them, but simulations are rather costly to run, especially in 3D, and this limits the number of possible independent realisations of these experiments. Recently, L. Chevillard, J-B Durrive and myself have been developing new models of synthetic turbulence [2,3] : we can produce a realisation of our synthetic random flows at a computational cost no more than the equivalent of a few time-steps of a simulation. These synthetic flows bear the classical signatures of intermittency (power spectra and non-Gaussian statistics of increments) and reproduce other known correlations. However, these early models fail at reproducing coherent structures seen in DNS [4,5]. This is a hint that the increment statistics (or equivalently the structure functions and multifractal spectrum) do not constrain sufficiently the coherent structures.

Here, I will present a set of new ideas for synthetic models which allow to generate some coherent features. These ideas are a set of approximations which are directly applied to the filtered Navier-Stokes equations. As such, they are straightforward to generalise to many types of turbulent flows (2D/3D, MHD, compressible or not, gravitating..). The purpose of the present work is to quantitatively assess their validity in the most simple framework : 2D decaying incompressible hydrodynamical turbulence. To this aim, we will compare results of 2D decaying simulations to snapshots of synthetic turbulence generated with our method (MUSCAT, for MUlti-Scale Turbulent Synthesis). We will first assess quantitatively the difference by using the standard tools of increments statistics and show that they can be fairly well revovered. Then, we will use the metrics provided by scattering transforms [1], which are a more stringent tool to constrain non-gaussian features.

We hope such generative methods for turbulence can be used for a number of applications. They can simply serve as better initial conditions for developped turbulence. 3D synthetic fields could be adjusted to observed projections with the hope to recover the underlying structure in astrophysical images. Similar methods may be used to guess a turbulent field outside regions where it is measured, such as in weather forecast. Finally, they can also serve as simplified statistical models to theoretically understand measured statistical laws.

Références

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