

# Experimental Nonlinear Waves along a Torus of Fluid

Filip Novkoski<sup>1</sup>, Chi-Tuong Pham<sup>2</sup>, Eric Falcon<sup>1</sup>

<sup>1</sup> Université de Paris, MSC, UMR 7057 CNRS, F-75013 Paris, France

<sup>2</sup> Université Paris-Saclay, LISN, UMR 9015 CNRS, F-91405 Orsay, France

filip.novkoski@u-paris.fr

Curved interfaces such as toroidal drops are ubiquitous in nature, but are unstable, making them difficult to control and study. Using an original technique we create a stable and stationary torus of liquid, deposited on a superhydrophobic substrate allowing for a systematic study of waves along its inner and outer border under curved and periodic conditions [1,2]. By exciting the torus border and recording the displacements of the two borders, we study the dispersion relation of the torus, yielding a rich spectral structure: gravity-capillary waves, sloshing modes and a center-of-mass mode [1]. We will first show that nonlinear waves in form of solitons can propagate along the torus borders for sufficiently strong forcing. We will then stress the observation of subsonic elevation solitons which are due to a strong influence of periodic boundary conditions through a Korteweg-de Vries equation, giving a non-trivial dependence of the soliton velocity on its amplitude and torus curvature. Finally, we will discuss the observation of a triadic resonance instability between two different dispersion branches, namely the sloshing and gravity-capillary modes [3]. Such nonlinear interactions thus lead to a transfer of energy between the two branches which is fully characterized. For a stronger forcing, additional waves are generated by a cascade of three-wave interactions populating the wave spectrum. Such a three-wave two-branch interaction mechanism is probably not restricted to hydrodynamics and could be of interest in other systems involving several propagation modes.



**Figure 1.** A stable torus of fluid of outer radius  $R_o = 4.8$  cm on a superhydrophobic substrate.

## References

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