Probabilistic study of the formation of extreme waves

Miguel Garrido[1], Ricardo Grande[2], Kristin Kurianski[3], Gigliola Staffilani[4]

[1] Department of Statistics, Columbia University, New York, NY, USA
[2] Département de mathématiques et applications, École Normale Supérieure, CNRS, PSL University, 75005 Paris, France
[3] Department of Mathematics, California State University Fullerton, Fullerton, CA, USA
[4] Department of Mathematics, Massachusetts Institute of Technology, Cambridge, MA, USA
ricardo.grande@ens.fr

Oceanographers generally define extreme waves to be water waves whose amplitude exceeds twice the characteristic wave height expected for the given surface conditions. There are several open question regarding their formation :

- *Formation mechanism :* What is the most likely way in which such waves form ?
- *Profile :* What is the typical shape of such waves ?
- *Forecast :* what are the most likely initial conditions that will produce a extreme wave in the future ?

Some of these questions are very difficult to answer in the deterministic setting due to unusual, pathological solutions which might not be relevant in most practical situations. The statistical approach we propose allows us to avoid such issues.

In [1], we use the cubic NLS equation as a toy model for the propagation and interaction of waves in a large one-dimensional box of sufficiently large size with periodic boundary conditions. We consider random initial data, and we set out to study the set of waves that reach an amplitude of at least $z > 0$ at time $t > 0$, namely

$$\mathcal{A}(t,z) = \left\{ \sup_{x\in\mathbb{T}} \ \eta(t,x) \geq z \right\} \ , \tag{1}$$

where $\eta(t,x) = |u(t,x)|$, and u solves the focusing cubic NLS equation on a large periodic box. Our goal in this setting is to give a simplified, statistical description of this set as $z \to \infty$, which is supported by numerical and experimental evidence in a series of works by Dematteis, Grafke, Onorato and Vanden-Eijnden [2,3].

We set ourselves in a simplified setting where the initial data are statistically small $(\mathbb{E}|\eta_0(x)|^2)^{1/2} \approx \varepsilon$, which is mathematically equivalent to a weakly nonlinear problem.

Our main result in [1] is a large deviations principle for the set $\mathcal{A}(t,z)$ as $z \to \infty$. This result is obtained by identifying a small subset of $\mathcal{A}(t,z)$ that concentrates all the probability of $\mathcal{A}(t,z)$ as $z \to \infty$.

This subset is a small neighborhood around the most typical profile of such extreme waves, which is the nonlinear evolution of an explicit choice of initial data. This yields a statistical characterization of the most typical profile of these waves as well as the initial datum that created them.

In an upcoming work [4], we extend our techniques to tackle the case where η in (1) solves the (periodic) Water Wave system.

Références

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