Wave scattering and irreversible wave capturing by two-dimensional turbulent flow

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Coupled wave-flow systems are prone to a wide variety of phenomena. Recently, several studies (e.g. [2] and [5]) have shown that, in presence of a weak and stationary random mean flow, dispersive waves whose dispersion relation $\omega(k)$ is a power law in k, may exhibit a "scattering" behaviour. In agreement with theoretical surveys [3], said scattering lies on the dispersion relation map in the phase space: the wave vector $\mathbf{k}$ is rotating while its magnitude remains constant (and, consequently, so does the frequency). This process has been proposed for internal waves as well [4] but, due to the particular nature of their dispersion relation, such result does not reflect the capability of the group velocity to evolve without changing the frequency, potentially breaking the fundamental assumption of a weak mean flow compared to the group velocity. In this work, we explore the elementary set of conditions that allows to obtain a scattering only in wave number orientation or both in wave number orientation and magnitude. We show that two regimes coexist with a non-trivial transition from one to the other.

Starting from a set of PDEs describing dispersive waves in presence of a turbulent mean flow, we first present a procedure using the Wigner transform to reduce the system to the following set, constituted of a Liouville equation on the energy density A with a Hamiltonian Ω involving a Doppler shifting term

$$\partial_t A + \{\Omega, A\} = 0, \tag{1}$$

$$\Omega = \omega + \epsilon_0 \mathbf{U} \cdot \mathbf{k}, \quad \epsilon_0 \in \mathbb{R}^+, \tag{2}$$

$$\omega = |k|^\alpha, \quad \alpha \in \mathbb{R}. \tag{3}$$

We identify two relevant parameters for our study: α, controlling the power law of the dispersion relation; and ϵ_0, indicating the initial ratio between the average value of the stationary mean flow and the initial group velocity. We then investigate this phase diagram thanks to a 2D ray tracing scheme in order to identify the different asymptotic regimes. This scheme is derived from the Hamiltonian system

$$\partial_t \mathbf{x} = \epsilon_0 \mathbf{U}(\mathbf{x}) + \mathbf{c_g}(\mathbf{x}, \mathbf{k}), \tag{4}$$

$$\partial_t \mathbf{k} = -\epsilon_0 \mathbf{k} \cdot \nabla_{\mathbf{x}} \mathbf{U}(\mathbf{x}). \tag{5}$$

We demonstrate that there is a range of dispersive systems for which the frequency stays constant with a scattering only observed in wave number angle. However, for other systems the assumption that the group velocity is larger than the mean flow can break, yielding to a drift in frequency. This scattering is associated to an irreversible wave capturing process by the turbulent flow, the rays being trapped in large scale features. We also show that the existence of these regimes is robust by testing different turbulent flows and that the transition between them is modified when the mean flow becomes time-dependent.

References

1. Boury, S., Bühler, O., and Shatah, J. Wave scattering and irreversible wave capturing by two-dimensional turbulent flow. In preparation for *PRE*.
2. Dong, W., Bühler, O., and Smith, K.S. 2020. Frequency diffusion of waves by unsteady flows. *JFM*, 905:R3.
3. Ryzhik, L., Papanicolaou, G., and Keller, J.B. 1996. Transport Equations for Elastic and Other Waves in Random Media. *WM*, 24:327–370.
4. Savva, M.A.C., Kafiabad, H.A., and Vanneste, J. 2021. Inertia-gravity-wave scattering by three-dimensional geostrophic turbulence. *JFM*, 916:A6.
5. Villas Bôas, A.B. and Young, W.R. 2020. Directional diffusion of surface gravity wave action by ocean macroturbulence. *JFM*, 890:R3.